

Copy /

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

7N-73-TNL
140213
518

DIFFRACTION OF MOLECULAR BEAMS

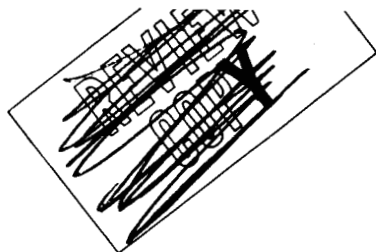
By I. Estermann and O. Stern

Translation of "Beugung von Molekularstrahlen"
Zeitschrift für Physik, vol. 61, 1930. pp 95-125

(NASA-TM-101093) DIFFRACTION OF MOLECULAR
BEAMS (NASA) 51 p

N88-70813

Unclas
00/73 0140213



NASA FILE COPY

Open copies on test
and stored in file drawer
PLEASE RETURN TO
DIVISION OF RESEARCH INFORMATION
MATERIAL SERVICES
AND STUDY COORDINATION
WASHINGTON 25, D. C.

LANGLEY AERONAUTICAL LABORATORY

April 1958

DIFFRACTION OF MOLECULAR BEAMS

By I. Estermann and O. Stern

ABSTRACT

The results of some famous experiments made to verify the underlying (De Broglie) postulate of the quantum theory are presented. This work takes on new significance, however, because hypersonic flows of rarified gases tend to resemble the beams of atoms described in the report and the interaction of those beams with crystals may be considered as a new model for the gas-surface interaction problem.

INDEX HEADINGS

Flow, Slip	1.1.5.1
Flow, Free-Molecule	1.1.5.2

DIFFRACTION OF MOLECULAR BEAMS*

By I. Estermann and O. Stern

If a molecular beam (H_2 ; He) falls on a cleavage surface of a crystal (LiF), the beams scattered by this surface show, in all details, an intensity distribution which corresponds to the spectra produced by a cross grid. For different m and v the wavelength calculated from the grid constant of the crystal has the value required by De Broglie, $\lambda = \frac{h}{mv}$.

The following report contains a description of the tests which were made at the Institute here during the current year for proving the wave character of molecular beams predicted by De Broglie. For this purpose, the reflection and the scattering of molecular beams of helium or hydrogen by a cleavage surface of a crystal were investigated. The test setup was the same as in the report of Knauer and Stern;¹ however, a number of changes were made in the apparatus which will be described below, at the appropriate point. Part I contains the tests carried out by Stern from January to April;² a brief report on the results of these tests has already been published in the Naturwissenschaften.³ Part II presents the tests performed by both authors together.

*"Beugung von Molekularstrahlen." Untersuchungen zur Molekularstrahlmethode aus dem Institut für physikalische Chemie der Hamburgischen Universität No. 15. (Investigations concerning the molecular-beam method from the Institute for Physical Chemistry of the University at Hamburg, No. 15.) Zeitschrift für Physik, vol. 61, 1930, pp. 95-125.

¹Knauer, F., and Stern, O.: U. z. M. No. 11, Zeitschrift für Physik, vol. 53, 1929, pp. 779-791.

²The tests represent a continuation of the work of Knauer and Stern. Since Mr. Knauer wanted to discontinue his further collaboration in the tests because of his own investigations, Stern had to continue the tests, at first, by himself.

³Stern, O.: Naturwissenschaften, 17, 1929, p. 391.

PART I

Although the tests described here are much outdated by the tests represented in part II, they will be briefly outlined at this point because they contain the first certain proof of the occurrence of diffraction phenomena on a cross grid in the case of molecular beams. The tests of Knauer and Stern regarding the dependence of the reflectivity on the crystal orientation formed the starting point. At that time, the two orientations drawn in figures 1 and 2 were investigated. In the "straight" position (fig. 1), one of the two principal axes lying in the crystal surface is perpendicular to the plane of incidence. For the other position (fig. 2), the crystal was rotated in its plane by 45° . In what follows this position of the crystal, in which a principal axis of the surface grid of ions of the same polarity is perpendicular to the plane of incidence, will be designated as the 0-position. The angles of rotation of the crystal in its plane are counted from that position and are denoted briefly below as "rotation." Thus, figure 1 corresponds to a rotation of 45° .

The apparatus was rebuilt in such a manner that, not only could the two positions mentioned be investigated, but the crystal could be continuously rotated in its plane with the aid of a gear and an endless screw to be actuated by a polished stopper. The result of this test for an angle of incidence of 10° ⁴ is given by curve I (fig. 3). The reflectivity has a maximum in the two positions investigated before (rotations 0° and 45°). The sharpness of the maximum for 0° is considerable; a rotation of $7\frac{1}{2}^\circ$ reduces the reflectivity by one-half. This behavior prompted the conjecture that for the zero position, in addition to the reflected beam, diffracted beams penetrated into the slit of the receiver which at a slight twist of the crystal lattice pronouncedly changed their position so that they no longer fell into the receiver slit. Theoretically, such diffraction maxima are to be expected if the cross-grid effect does not come from a grid built up of both positive and negative ions, as had been assumed in the report of Knauer and Stern, but only from a grid of ions of the same polarity. The theory (see appendix) shows that in the latter case diffraction spectra should occur which, for the De Broglie wavelengths to be expected, form an angle of about 8° to 9° with the reflected beam and lie almost in the beam plane (within 1° to 2°). The "beam plane" is understood to be the plane which is determined by the direction of the beam and the long edge of the rectangle forming the cross section of the beam (5×0.5 mm). In these tests, the

⁴The angle of incidence is always understood to be the angle between the incident beam and the surface of the crystal (glancing angle); small angles of incidence therefore signify a flat incidence.

beam fell onto the crystal surface in such a manner that the long edge of this rectangle was parallel to the crystal surface. (Cf. figs. 1 and 2.) For the dimensions of the apparatus used (height of the receiver slit 5 mm, width 0.5 mm, distance from the crystal 15 mm (see fig. 4)), such maxima also had to get into the receiver. The theory yields, furthermore, that in the case of a slight rotation of the crystal, the diffraction spectra mentioned partially shift out of the beam plane, or, for further rotation, possibly are not produced at all. Also, according to the theory, this effect becomes weaker for larger angles of incidence. Curve II (fig. 3) is in agreement with this; it was obtained with an angle of incidence of 20° and shows considerably flatter maxima. This assumption explains at the same time the noteworthy result of Knauer and Stern that for a low beam temperature (100° K) a maximum of reflectivity was found at an angle of incidence of about 20° . At this temperature, due to the larger De Broglie wavelength, the diffraction maxima are obtained only at an angle of incidence of approximately 20° . In order to make a direct investigation of these diffraction spectra possible, the apparatus was rebuilt so that the beam fell onto the crystal "up-end"; the beam plane therefore now coincided with the plane of incidence (fig. 5). This was attained by then fastening the crystal horizontally to the crystal support, whereas it had been attached vertically before (cf. figs. 6 and 7); correspondingly, the beam was inclined by the angle of incidence, $11\frac{10}{2}$ ($1:5$). The receiver was inclined by the same angle but otherwise maintained its position so that its axis of rotation was then perpendicular to the crystal surface, whereas formerly it lay in the plane of the crystal surface. Curve III (fig. 8) is the first curve obtained with this arrangement. It actually shows the expected diffraction maxima. That these latter did get into the receiver at all, although they lie more closely to the crystal surface than the reflected beam (see appendix, p. 17), is explained by the considerable height of the receiver slit (5 mm). That in reality the diffraction maxima is dealt with here is shown by curve IV (fig. 8), which was obtained with a crystal rotated in its plane by 45° . The theory states that for this position of the cross grid, no diffraction spectra lying so closely to the reflected beam occur; in agreement with the theory, curve IV does not show any indication of this. Curves V, VI, and VII (fig. 8) are obtained with helium at high and low temperatures and with hydrogen at high temperature, and show the same behavior.⁵ The shifting of the diffraction maxima with the temperature, to be expected according to De Broglie's formula $\lambda = \frac{h}{mv}$, is present as far as direction is concerned but is considerably too small. The insufficient range of

⁵Sometimes the same crystal was used on several days. In this case it was conserved by filling the apparatus with helium of 1 to 2 mm pressure. If the crystal is left in the vacuum for any appreciable time, its surface is ruined.

adjustment could have been the reason for this, as has already been mentioned in the note in "Naturwissenschaften"; for a more detailed discussion, compare part II. Anyway, these tests prove that the phenomena observed resulted from diffraction by a cross grid.

PART II

First, some of the tests described in part I were repeated, with the same results. As was pointed out above, it became clear that the range of adjustment was inadequate.

ADJUSTMENT

Adjustment of the Furnace Slit

When the apparatus was heated, the furnace slit was evidently shifted downward, due to the expansion of the gas supply tube carrying it; when the apparatus was cooled, the slit was shifted upward, due to contraction of this tube. A rough calculation yields approximately 0.5 mm as the amount of this shift. It became noticeable, also, by the fact that fluctuating values were obtained for the intensity of the reflected beam at high or low temperature, according to the adjustment. In order to compensate for this expansion and simultaneously to improve the adjustment during the test, part of the supply tube was replaced by an elastic capsule D. Thereby it was possible to shift the furnace slit in the direction of its height during the test in the manner shown in figure 9 with the aid of the screw Sch actuated by the polished stopper S. Lateral shifting was possible, as previously, by rotating the stopper carrying the entire furnace slit. For tests with a cooled beam, an additional tube R was built in, which was filled with liquid air and could be connected with the furnace slit by a flexible braided copper cord. The furnace slit had, at first, a length of 1 mm; then a length of 0.5 mm and a width of 0.2 mm.

Adjustment of the Crystal Support

It happened that, when the crystal was rotated in its plane by 90° , the reflected beam was shifted by several degrees - an indication that the reflecting crystal surface was not perpendicular to the axis of rotation. This resulted because the crystals sometimes were not quite parallel on the bearing surface. The crystal support was therefore modified so that the bearing surface was fastened in a manner similar to a Cardan

suspension (fig. 10). With the aid of the screws S_1 and S_2 which were rotated from the outside by two polished stoppers provided with screwdriver slots, the crystal surface could be rotated about two axes perpendicular to one another and to the axis of rotation. In this manner the crystal surface could be adjusted perpendicular to the axis of rotation by observation of the reflected beam in the various positions distant from one another by 90° ; this was possible within an accuracy of less than 0.5° . The crystal was attached to the support in such a manner that the principal axes of the surface grid of ions of the same polarity were parallel to the axes of rotation of the Cardan adjustment. For heating the crystal, a strip of thin platinum sheet, insulated with mica, was fastened to the lower side of the bearing surface; this strip was heated electrically.

The Receiver

In order to correctly observe the first investigated cross-grid spectra of the order 0.1, the receiver would have had to be rotatable about the principal axis of the surface grid of ions of the same polarity which lay in the plane of incidence. (See appendix, p. 17.) For the existing apparatus, this would have been attainable only with extreme difficulty. In the apparatus used in this test, the axis of rotation of the receiver coincided with that of the crystal support. Thus, the receiver would have had to be provided with an additional moving mechanism in order to make variations of its height above the crystal surface possible. It would have been necessary to make sure that the direction of the receiver channel always pointed toward the point of intersection of the axis of rotation of the crystal support and the crystal surface. This, also, would have been attainable only at the price of great mechanical complications. As mentioned before, this difficulty was avoided in the previous tests in a crude manner by making the receiver slit so high that beams starting from the crystal, even at a rather flat angle, could still get into the slit. In order to obtain cleaner conditions and, simultaneously, to avoid the mechanical complications mentioned before, a receiver slit of smaller height (1.5 mm) was used and the possibilities of motion of the new crystal support were utilized. Instead of bringing the receiver up to the crystal surface, the crystal was tilted about the axis perpendicular to the plane of incidence of the beam. The requirement that the receiver channel always point to the point of intersection of the crystal support and the crystal surface was then automatically fulfilled. It is true that the angle of incidence was increased, but only by a few degrees. As will be shown by the investigations of the dependence of the reflectivity on the angle of incidence, which is reported on below, the reflection is practically independent of the angle of incidence in the range considered (angle of incidence up to 20°) for the test conditions. It is assumed the same for the

diffraction. The tests were performed in such a manner that, at every position of the receiver, a search was made for the "most favorable tilt," that is, the tilt at which the intensity of the diffracted beam attains its maximum. (Cf. also the discussion on the influence of the tilt in the appendix, p. 19.) Another small change must be mentioned: The long helical springs in the manometers were replaced by short elastic hooks; thereby the sensitivity to shocks of the manometers was reduced.

DIFFRACTION TESTS WITH SODIUM CHLORIDE

First of all, the tests with helium and sodium chloride were repeated with the improved apparatus, at first not at the most favorable tilt but at constant tilt. The results may be seen from curves VIII to X (fig. 11): angle of incidence, $11\frac{1}{2}^{\circ}$ + tilt angle; crystal orientation, a rotation of 0° . For curve VIII (tilt angle 0°) it is noted that the reflected beam is strong, while the diffraction maxima are relatively weak. For curve IX, tilt angle $3\frac{1}{2}^{\circ}$, conversely, only a trace of the reflected beam can be seen, whereas the diffraction maxima are much more pronounced and shifted toward the outside. According to the elementary theory of cross grids (see appendix) this behavior is to be expected, since the diffraction maxima lie more closely to the crystal than the reflected beam. The spread of the diffraction maxima in the case of strongly tilted crystal is also required by the theory. A further proof for the fact that the diffraction maxima stemming from a cross grid are dealt with here may be seen from curve X for which the crystal had been rotated in its plane by about 10° . By this rotation one maximum is reinforced, the other weakened (see appendix, p. 20) for constant tilt.

Curves XI and XII (fig. 12) reproduce the results of a test with most favorable tilt (angle of incidence $11\frac{1}{2}^{\circ}$ + tilt angle, beam temperature for curve XI 290° K, for curve XII 580° K). In agreement with the theory, the angles of most favorable tilt increase with the distance from the reflected beam. Table 1 shows this behavior in the above test; it was verified also in all following tests.

⁶The tilt angles are estimated only from the dimensions of the crystal support; their absolute value is uncertain up to 20 percent. The relative accuracy amounts to about $\frac{1}{4}^{\circ}$.

The calculated distance of the cross-grid spectra from the mirrored beam results from table 2.

For the test described in part I, the position of the maximum found at room temperature agreed with the calculated one for the order 01. The temperature shift of the maxima was considered too small even then; it was emphasized that this result was not certain because of inadequate adjustment. The tests made with the improved adjustment and flat angle of incidence, $11\frac{1}{2}^\circ$, (curves VIII, IX, XI, and XII) yielded, at room temperature, the same result for constant angle of incidence as well as for the most favorable tilt. The temperature shift, however, was practically 0; also for 580° K the maximum lay approximately at 9° .⁷ It is assumed that the reason for this is that for sodium chloride the maxima of the order 02 also are very intensive. For a flat incidence ($11\frac{1}{2}^\circ$), the maximum of the order 02 would be suppressed for geometrical reasons (see appendix, p. 16) at room temperature; at a high beam temperature, in contrast, it would be clearly noticeable.

In order to prove this assumption, tests were made with a steeper angle of incidence ($18\frac{1}{2}^\circ$; 1:3) at three temperatures (curve XIII 100° K , curve XIV 290° K , and curve XV 580° K (see fig. 13)), and, especially, the curve for room temperature (XIV) was measured with particular care (distance of the test points 1°). For this angle of incidence, the maximum of the order 02 should still be noticeable at room temperature, also. Actually, curve XIV does have the character of a curve resulting from superposition of the two orders. Curve XV, obtained at high temperature, shows the maximum at the point where it is to be expected for the order 02. Thus it appears as if, with rising temperature, the intensity of the second order would increase in comparison to the first. Conversely, for curve XIII, 100° K , the maximum is approximately at the point to be expected for the first order, since here the second order is so far distant from the mirrored beam (29°) that it is greatly weakened, if only for geometrical reasons. Theoretically, it ought to be perfectly reasonable that the ratio of the intensities of the different orders should vary with the temperature, because this ratio is known to be greatly dependent on the form of the grid lines. The helium atoms with a higher velocity will penetrate more deeply into the potential field of the crystal than the slower ones.⁸ It is therefore quite

⁷Since the position of the mirrored beam, particularly because of its shape, could not be determined more accurately than within about 1° , the mean distance was always measured between the maxima on both sides.

⁸This conclusion from the classical theory is maintained in wave mechanics also.

conceivable that the "form of the grid lines" is different for helium atoms of different velocities. If this should be true, investigation of the intensity ratio of the different orders would provide a means for learning something about the potential variation at the surface of a crystal. Of course, this interpretation of the test results is still completely hypothetical and would have to be checked by tests with "monochromatic" molecular beams (molecular beams of a single velocity). Such an investigation will probably no longer offer any particular experimental difficulties and is being started. For the time being, a more detailed investigation has not been made of the situation in the case of sodium chloride, which is evidently somewhat complicated, since it has meanwhile been found out that tests on LiF grids yield a much simpler and clearer picture. Still, it should be emphasized once more that the interpretation of the observed phenomena as diffraction of the De Broglie waves of the molecular beam by the cross grid of the crystal surface of NaCl seems to be completely certain.

DIFFRACTION TESTS WITH LITHIUM FLUORIDE

A considerably better reflection, sharper beams, and more intensive diffraction maxima were obtained in tests with LiF.⁹ Also, the diffuse subsurface which was still perceptibly present in the sodium chloride tests was now practically unnoticeable.

Curves XVI to XIX (figs. 14 to 16) and table 3 contain the results of the tests with an angle of incidence of $11\frac{1}{2}^{\circ}$ and the most favorable tilt. The calculated positions of the maxima are marked in the curves by arrows. The agreement between calculation and experiment lies completely within the test accuracy of $\frac{1}{2}^{\circ}$ to 1° , with the exception of the values for hydrogen at 290° for which the maximum for the "most probable wavelength," $16\frac{3}{4}^{\circ}$, lies outside the angle of incidence, $11\frac{1}{2}^{\circ}$ + tilt angle. However, the tilt angle is limited; it must not be larger than half the angle of incidence because, otherwise, the crystal surface would shut out the diffracted beam. Actually, the tilt angle cannot even be made that large, due to the finite expansion of the beam; in the case above, the tilt angle cannot be made larger than 4° . The maximum found at $14\frac{1}{2}^{\circ}$ is thus simulated by the decrease at large angles

⁹Artificial crystals, manufactured by R. Pohl, Göttingen, were used. The authors would like to express here, also, their heartfelt gratitude to Mr. Pohl for his kindness in letting them have his entire stock of LiF crystals.

(long waves). The curve reproduced later, with a larger angle of incidence, shows the maximum at the correct point.

Curves XX to XXVI (figs. 17 and 18) and table 4 contain the results for an angle of incidence of $18\frac{1}{2}^{\circ}$ and the most favorable tilt.

Curve XX (fig. 17) gives a complete curve as an example; in curves XXI to XXVI (fig. 18) there is always one-half of the measured curve shown, with omission of the mirrored beam. For helium of 100° K, the maximum has already been found at angles that are too small; this is again to be explained by the fact that, for the most probable wave length λ_m , the diffraction maximum lies outside the angle of incidence. It is remarkable that, in all cases where the deviation from the calculated value is larger than $\frac{1}{4}^{\circ}$, the maximum is found to lie at angles smaller than the calculated ones. This is caused by the fact that for various reasons (see the following section), the intensity is measured somewhat too small for large angles. Thereby the maximum of the curve is shifted somewhat toward smaller angles. It may therefore be said that in all cases the agreement between observed and calculated values lies completely within the test accuracy for the larger angle of incidence as well. The diffraction maximum of helium at room temperature, for instance, lies at the same point as that of hydrogen at double the absolute temperature. The curves XXI to XXVI thus give a complete confirmation of De Broglie's relationship

$$\lambda = \frac{h}{mv}$$

with respect to the dependence of the wavelength on m and v as well as with respect to the absolute values themselves.

So far, it has been tacitly assumed that the intensity distribution in the diffraction maxima corresponds to the Maxwell distribution of the velocities or wavelengths in the molecular beam. A comparison of the diffraction curves of the present tests with the Maxwell distribution in the beam represented in wavelengths (curve XXVII, fig. 19) shows that this assumption is true, generally speaking. It appears, however, that the intensity decreases somewhat too strongly for large angles (long waves). This has the following reasons: The dispersion increases with growing distance from the mirrored beam. The angle β (see appendix, p. 16) is given for the first order by

$$\cos \beta = \frac{\lambda}{d}$$

thus

$$d\beta = \frac{-1}{d \sin \beta} d\lambda$$

that is, the wavelengths contained in the wave range $d\lambda$ are distributed over a range of angles $d\beta$ which increase with decreasing $\sin \beta$, thus also decreasing β . For small diffraction angles $R - \beta$, β is almost 90° ; thus $\sin \beta$ is almost 1. For the diffraction angles up to about 20° which are of interest, the values of $\sin \beta$ are

$R - \beta$, deg	β , deg	$\sin \beta$
0	90	1.0
5	85	.9962
10	80	.9845
15	75	.9659
20	70	.9397

All of them are still very nearly equal to 1; even so, this influence causes a too rapid decrease of the intensity toward larger diffraction angles. The second reason lies in the test arrangement, that is, in the fact that, for larger angles, one can no longer see from the receiver the entire surface of the crystal "illuminated" by the molecular beam. This influence would have to be calculated from the dimensions of the apparatus. It will be sufficient to indicate an upper limit for it. The area seen from the receiver is at least equal to the area seen at 0° multiplied by the cosine of the diffraction angle $R - \beta$. This reduction is therefore, at most, as large as that due to the dispersion; the total reduction by the two influences could therefore, at most, amount to 12 percent, for a 20° diffraction angle. Experimentally, the reduction seems to be, in part, still larger. It would, after all, be possible that a real effect is involved here, perhaps in such a manner that the long waves (slow molecules) do not penetrate sufficiently deeply into the potential field of the grid, that therefore the grid lines are, as it were, too flat for them. A more detailed discussion of these effects will be postponed until the above-mentioned investigations with monochromatic molecular beams have been carried out.

A few more curves (XXVIII to XXX, fig. 20) are shown with constant tilt at tilt angles of 0° , 3° , and 6° , for which it can be seen very well how the mirrored beam disappears with increasing tilt angle, while the diffraction maxima become much stronger and shift outward. In

curve XXXI (fig. 21), still another test is presented with a crystal rotated in its plane by 8° ; in this curve the pertaining tilt angle is noted for each test point. It can be seen clearly how on one side a pronounced maximum is obtained at the correct point (12°) without any considerable tilt, while on the other side, in spite of the maximum tilt, only traces of a maximum remain visible. As has been remarked already, in connection with the corresponding sodium chloride tests, this behavior corresponds in all details to the theory.

DIFFRACTION SPECTRA IN THE PLANE OF INCIDENCE

After it had been found that LiF has a surface grid which is much more suitable for these tests than that of NaCl, the types of spectra which Knauer and Stern had investigated for sodium chloride were also investigated for LiF. Those old tests had not given any clear results, not only because of the bad properties of the NaCl, but also because, as has been explained in part I, due to the receiver slit being too high, the diffraction maxima investigated above, which lay beside the mirrored beam, in part also got into the receiver. Therefore, the height of the receiver was reduced to 1 mm for these tests; this was quite sufficient, especially since, for LiF, the diffraction maxima mentioned lie farther away from the beam due to the smaller grid constant. Aside from this, the arrangement drawn in figure 6 was used again. The orientation of the crystal was such that the plane of incidence was parallel to a cube face of the crystal; the angle was thus halved between the principal axes of the surface grid of ions of the same polarity (rotation 45° , cf. p. 2). The diffraction spectra therefore are of the order +1, +1, or -1, -1. They correspond to the spectra of a line grid with the grid constant $\frac{d}{\sqrt{2}}$. (See appendix, p. 21.)

Thus, these spectra are much simpler than the cross-grid spectra investigated above. Since they lie in the plane of incidence, the complication due to the tilt of the crystal is eliminated. Nevertheless, a much more thorough investigation of the cross-grid spectra was considered desirable because they are much more intensive and much cleaner than the line-grid spectra. The reason is evidently that in the case of cross-grid spectra the incident beam as well as the diffracted beams form only small angles with the crystal surface; thus, the "roughness" of the surface (cf. the considerations in the section on reflection) is not too disturbingly noticeable. For a rotation of the crystal by 45° (thus for the position denoted as rotation 0), the corresponding spectra (in this case, of the order 01) were just barely perceptible. In curves XXXII to XLI (fig. 22) the results are given for helium and the beam temperatures 100° and 290° K for angles of incidence from 10° to 70° . The arrows give the calculated positions of the diffracted

beam for λ_m . As can be seen, the agreement between the calculated and observed positions lies generally within the observation accuracy which in this case, since measurements were made only at every 5° , is approximately 2° to 3° .

Only for short waves and large angles of incidence are the measured maxima in a position which is, compared to the calculated maxima, shifted toward the right (larger angles) by up to 7° . The fact that these deviations occur only at large angles of incidence and even then only for short waves (beam at room temperature) shows clearly that evidently the roughness of the surface already plays a part here; the extraordinarily bad reflection supports this opinion. The direction of the shift would indicate that the long waves are favored while the opposite is true for the short waves. Regarding the curves for 290° K , it should be noted also that for angles of incidence below 45° , in agreement with the theory, no "negative" maxima (angle of diffraction smaller than angle of reflection) were found. Thus, all observations are in perfect agreement with the theory for these spectra as well.

REFLECTION

The reflection of the molecular beams of He from LiF obeys the law of reflection within the test accuracy (in this case about $\frac{1}{4}^\circ$). This applies also for the other cases investigated (H_2 from LiF, H_2 and He from NaCl); however, the reflection is not as strong in these cases and the beam is somewhat indistinct. The values of the reflectivity (maximum intensity in the reflected beam divided by maximum intensity in the direct beam) vary somewhat, of course, according to the quality and the age of the crystal cleavage surface; however, the order of magnitude is always reproducible. Quite generally speaking, the reflectivity depends strongly on the angle of incidence and the orientation of the crystal.

Dependence on the Angle of Incidence

In all cases the reflectivity decreases with increasing angle of incidence. Compare, for instance, the curves XLII and XLIII in figure 23 (reflectivity of LiF for He for straight orientation of the crystal, rotation 45°). The most natural explanation is probably that the crystal cleavage surface represents a dull surface. For such a surface, the specular reflection becomes considerable only when the projection of the height of the roughnesses onto the beam becomes

smaller than the wavelength. The same behavior was found in the reflection of molecular beams from highly polished surfaces in the tests of Knauer and Stern wherein, corresponding to roughnesses of about 10^{-5} cm, angles of incidence of about 10^{-3} , that is, of a few minutes, had to be used in order to obtain the beginning of specular reflection. In the case at hand, conversely, from the appearance of very good reflection at angles of incidence up to 20° and a steep drop at larger angles, it should be concluded that there exist roughnesses of the order of magnitude of only a few wavelengths (2 to 3), thus about 1\AA . This is the order of magnitude of the amplitude of the temperature oscillation of the grid ions. This interpretation is supported by the increase in reflectivity with decreasing temperature of the crystal.¹⁰ Quite independently of this assumption regarding the character of the roughnesses, the increase in reflectivity with decreasing beam temperature (longer De Broglie waves), which can be seen from the curves, seems to favor the interpretation as a reflection from a dull surface. It is true that the observations of the authors of this paper do not show this increase in all cases; even the inverse behavior occurs sometimes. However, this result need not absolutely constitute a contradiction of the above assumption, since the latter refers to the entire radiation scattered in phase by the crystal surface, and therefore contains not only the directly reflected beam but also the diffracted beams. Unfortunately, this total intensity cannot be determined, since all diffracted beams could not be investigated. A decision concerning the admissibility of the authors' assumption is, of course, possible only on the basis of such measurements; nevertheless, it will be regarded, for the time being, as a natural working hypothesis which is compatible with the experimental results. If this assumption should be confirmed, the amplitude of the temperature oscillations of the ions could be measured in a simple manner, the existence of a zero-point energy, for instance, could be proved. (Cf. the corresponding tests with X-rays.)

Dependence on the Crystal Orientation

First of all, it was almost always found that the reflection was considerably larger (frequently more than twice as large) for a straight position of the crystal (rotation 45°) than for oblique positions (rotation 0°). This seems to indicate that at the oblique position, so much of the intensity scattered in phase goes into the diffraction maxima of the order 01, which are here particularly intense, that not much is left for the mirrored beam. For the straight position of the crystal these

¹⁰Cf. Knauer and Stern, l.c. The authors of the present report have reproduced this result repeatedly.

maxima are not present, but only the considerably weaker ones of the order 11. For hydrogen, for which the intensity of the diffraction maxima is very much smaller, the difference in reflectivity in the two positions also is much less. Here, also, a certain interpretation is possible only after measurement of all diffraction spectra; however, in the mirrored beam and the diffraction spectra investigated, for a flat incidence (up to 20°) and for LiF and He, about half the total intensity of the incident beam is already contained, so that - since some diffuse scatter is certainly present as well - not much intensity is left available for the remaining spectra.¹¹ The dependence of the reflectivity on the orientation of the crystal seems to be rather complicated; curve XLIV in figure 24 (He from LiF, angle of incidence $11\frac{1}{2}^\circ$, beam temperature 290°K) gives an example for this. However, the possibility must be considered that this variation, especially the large changes in intensity at small angles of rotation, is influenced in addition, by diffraction maxima getting into the receiver slit, in analogy to curves I and II in part I. Since these diffraction spectra would have to lie very close to the reflected beam ($<3^\circ$), they would have to be produced by a grid with large grid constant. A grid of adsorbed molecules¹² could produce this condition, but of course other interpretations are possible also. A discussion will be useful only when more observations have been made.

Among the other crystals investigated were KCl and KBr. For He and KCl a reflection of the same order of magnitude as for sodium chloride was obtained, but a weaker diffraction; for KBr only a very weak reflection and no measureable diffraction. Among other gases investigated was neon, which does not show any perceptible regular reflection either from LiF or KCl.

The experiments described here need to be supplemented in many respects. Obviously, tests with "monochromatic" molecular beams will yield, in certain respects, much simpler results which can be interpreted more easily. The monochromatization will probably not offer any particular experimental difficulties. It may be achieved either by rotating gears or by "preseparation" with a second crystal. On the other hand, it would be very important for the continuation of the tests, to

¹¹This is, at the same time, good evidence that in these tests, no selection of wavelengths takes place, which also follows directly from the diffraction curves.

¹²It might also be possible that this cross grid consists altogether of adsorbed molecules; these latter would then have to agree with the LiF grid with respect to grid constant and arrangement.

investigated theoretically in more detail particularly the problem of the distribution of the intensity over the mirrored beams and the diffracted beams.

SUMMARY

The present tests have shown that a beam of He or H₂ molecules, which impinge on a cleavage surface of LiF, is diffracted by the latter as by a cross grid. Various orders of the diffraction spectra (0.1; 0, -1; 1.1, and -1, -1) were investigated; angles of incidence and orientation of the grid were varied. The De Broglie wavelength was varied by changing the temperature of the beam (variation of v) and changing the gas (variation of m). The results of the tests may be summarized as follows: If a beam of gas molecules impinges on a crystal cleavage surface, the reflected and scattered beams show an intensity distribution which corresponds in all details to the intensity distribution occurring in the diffraction of waves by a cross grid. If the distance between ions of the same polarity is substituted for the grid constant of the cross grid, the exact De Broglie value $\lambda = \frac{h}{mv}$ is obtained for the wavelength to be correlated to the beam.

Translated by Mary L. Mahler
National Advisory Committee
for Aeronautics

APPENDIX

CALCULATION OF THE CROSS-GRID SPECTRA

In considering a square cross grid, the x- and y-axes of the Cartesian coordinate system are placed along the two principal axes of the grid, and the zero point at the piercing point of the incident beam. The cross section of the latter is assumed to be infinitely small. If the incident beam forms the angles α_0 , β_0 , and γ_0 with the x-, y-, and z-axes, the direction of a diffracted beam is determined by the angles α , β , and γ which result from the equations

$$\cos \alpha - \cos \alpha_0 = h_1 \frac{\lambda}{d}$$

$$\cos \beta - \cos \beta_0 = h_2 \frac{\lambda}{d}$$

The symbol λ is the wavelength, d is the grid constant, and h_1 and h_2 are integers (order numbers of the spectrum). Every diffracted beam is thus the straight line of intersection of two cones about the x- or y-axis with the vertex at the zero point and the generating angles α and β .

First considered is the special case where the plane of incidence is the x,z-plane. Then

$$\beta_0 = 90^\circ, \quad \cos \beta_0 = 0, \quad \text{and} \quad \alpha_0 = R - \gamma_0$$

If $h_1 = h_2 = 0$, the reflected beam is obtained as the straight line of intersection of the α -cone, the cone about the x-axis with the angle α , where now $\alpha = \alpha_0$, and of the β -cone which in this case degenerates into the x,z-plane. If $h_1 = 0$, $h_2 = \pm 1$, the result is a spectra of the order 0, ± 1 , which was chiefly investigated. The diffracted beams then are the straight lines of intersection of the α -cone with the angle $\alpha = \alpha_0$ and of the β -cone with the angle β , whose numerical value results from the equation

$$\cos \beta = \pm \frac{\lambda}{d}$$

(See fig. 25.) The cones intersect only when $R - \beta < \alpha_0$. For easier visualization, the circles of intersection of the cones are considered with the unit sphere about the zero point projected on the y, z -plane parallel to the x -axis (fig. 26). The α -circle (circle of intersection between α -cone and unit sphere) is mapped, without change in size, as a circle about the zero point; the two β -circles ($h_2 = \pm 1$) become straight lines parallel to the z -axis. The guide beam from the zero point to the receiver, which is rotatable about the z -axis, cuts out of the unit sphere a circle parallel to the x, y -plane ("receiver circle") whose projection forms a straight line tangent to the α -circle at the point of intersection with the z -axis and parallel to the y -axis. The receiver is assumed to be adjusted - as it had been in the tests - in such a manner that it takes in the reflected beam at the position in the plane of incidence. As can be seen from figure 26, it would thus not be possible to measure the diffracted beam with this arrangement, since it lies more closely to the crystal than the reflected beam. In the first tests, this difficulty was avoided by working with a high receiver slit so that the receiver covered the strip. This can be seen from figure 26. For the tests in part II, the crystal was tilted about the y -axis, as mentioned in the text. The parallel projection drawn in figure 27 (the coordinate system is fixed to the crystal and is tilted with it) differs then from the former projection by the fact that the α -circle becomes larger, due to the increase of the angle of incidence by the tilt angle, and that the receiver circle is projected, instead of into a straight line, into a very flat ellipse which intersects the α -circle. For a given position of the receiver (rotation about the z -axis with crystal not tilted), the tilt angle can therefore be chosen in such a manner that the diffracted beam gets into the receiver. Because of the finite dimensions of beam and receiver, this angle is, of course, actually not quite sharply defined; as mentioned in the text, the procedure used was to seek every time that tilt angle which corresponded to the maximum intensity. Conversely, it is not the reflected beam that is measured if the receiver is rotated for constant tilt, but the diffracted beams, which are the farther distant from the reflected beam, the larger the tilt angle. (Cf. curves XXVIII to XXX, fig. 20.)

The projection of the unit sphere parallel to the z -axis is more suitable for the calculation of the diffraction angles. Then the representation known in X-ray spectroscopy as the "method of the reciprocal grid" is obtained. Figure 28 shows the special case where the incident beam lies in the x, z -plane. The points plotted are the projections of the piercing points of the diffracted beams with the unit sphere. Their coordinates are given by the equations

$$x = \cos \alpha = \cos \alpha_0 + h_1 \frac{\lambda}{d}$$

$$y = \cos \beta = \cos \beta_0 + h_2 \frac{\lambda}{d}$$

They form thus a grid with the grid constant λ/d , the "reciprocal grid." The cross-grid spectra investigated correspond to the points P_1 and P_2 ; they come about only when P_1 and P_2 lie within the equator circle. The projection of the receiver circle likewise becomes a circle about the zero point with the radius $\cos \alpha_0$. It is seen again that the points P_1 and P_2 do not lie on the receiver circle; that the diffracted beams thus are not caught by the receiver. Therefore, in order to measure the diffraction spectra given by the points P_1 and P_2 , the receiver would have to be adjusted in such a manner that the projection of the receiver circle yields the dashed circle. Then the angle ϑ_0 , which is measured between the plane of incidence and the plane determined by the z-axis and the diffracted beam, would be given by

$$\tan \vartheta_0 = \frac{\lambda}{d} \frac{1}{\cos \alpha_0}$$

As mentioned before, this simple case could not be realized because of the apparatus; instead, the crystal was tilted about the y-axis by the angle δ . The axis of rotation of the receiver then is no longer the z-axis but is inclined toward the z-axis by the angle δ (z'-axis), and the projection of the receiver is no longer a circle. The angle ϑ , measured between the plane of incidence and the plane determined by the z'-axis and the diffracted beam, is obtained by the equation

$$\tan \vartheta' = \tan \vartheta \frac{1}{\cos \delta - (\sin \delta) \sqrt{\tan^2(\alpha_0 + \delta) - \tan^2 \vartheta}}$$

where

$$\tan \vartheta = \frac{y}{x} = \frac{\lambda}{d} \frac{1}{\cos(\alpha_0 + \delta)}$$

since in the coordinate system x', y', z' fixed in space $\tan \vartheta' = \frac{y'}{x'}$, the above formula follows by substitution of the coordinates without primes. Since all occurring angles are small (maximum 20°), the approximation

$$\tan \vartheta' = (\tan \vartheta_0) \left[1 + \delta \tan \alpha_0 \left(1 + \sqrt{1 - \frac{\tan^2 \vartheta_0}{\tan^2 \alpha_0}} \right) \right] \quad 13$$

is sufficient for the purposes considered.

If the difficulty which stems from the fact that the receiver circle is not mapped as a circle is to be avoided, the stereographic projection, that is, the projection from a pole of the unit sphere onto the equator plane must be used. As is well known, in this projection all circles on the surface of the sphere are again mapped as circles. Figure 29 shows, as an example, this projection for LiF and He of 180° K, angle of incidence $18\frac{1}{2}^\circ$, tilt 3° . P gives the reflected beam, P_1 the diffracted beam. Because of the tilt, the receiver circle has an eccentric position; its center does lie on the x-axis - however, not at the zero point, but shifted toward the left. By performing the projection with consideration of the dimensions of beam and receiver slit, it was ensured that the measured "center of gravity" of the intensity yields, for practical purposes, that is, within $\frac{1}{4}^\circ$, the same diffraction angle which is obtained when the dimensions are neglected.

If the crystal is rotated in its plane by the angle ϵ , the following equations are obtained:

$$\cos \alpha_0 = \cos \alpha_{00} \cos \epsilon$$

$$\cos \beta_0 = \cos \alpha_{00} \sin \epsilon$$

¹³The tilt angle δ itself is given by the equation

$$\tan \delta = \frac{1 - \sqrt{1 - \tan^2 \vartheta_0}}{2 \tan \alpha_0}$$

according to a kind communication from Mr. W. Gordon. For the small angles appearing in these tests, the approximation $\delta = \frac{\vartheta_0^2}{4\alpha_0}$ is sufficient.

where α_{00} ($= \alpha_0$ for $\epsilon = 0$) is the angle between the incident beam and the crystal surface (glancing angle). For small angles of rotation ϵ the effect is that the diffraction spectrum on one side shifts closer to the receiver circle (P_1 in fig. 30, parallel projection parallel to the z-axis onto the x,y-plane with a coordinate system fixed to the crystal) and is therefore measured at small tilt angles, whereas the diffraction spectrum on the other side shifts from the receiver circle toward the equator circle and thus requires large tilt angles. Also, the spectrum breaks off here for much shorter waves than on the other side, because the grid points corresponding to the long waves already lie outside the equator circle. Figure 30 shows the case $\epsilon = 8^\circ$, which is measured as an example. For large ϵ , for instance $\epsilon = 45^\circ$, the diffraction spectra treated here no longer come about at all.

The calculation of the spectra lying in the plane of incidence shows that they correspond perfectly to the spectra of a line grid.

First case:- Rotation is 0° ; that is, a principal axis of the surface grid of ions of the same polarity lies in the plane of incidence. (Cf. p. 2.) Then there follows from the equations

$$\cos \alpha - \cos \alpha_0 = h_1 \frac{\lambda}{d} \qquad \cos \beta - \cos \beta_0 = h_2 \frac{\lambda}{d}$$

because of $h_2 = 0$, $\beta = \beta_0 = 90^\circ$

$$\cos \alpha = \cos \alpha_0 + h_1 \frac{\lambda}{d}$$

that is, the spectra of a line grid with the grid constant d .

Second case:- Rotation is 45° . Then $\alpha_0 = \beta_0$, $h_1 = h_2$, $\alpha = \beta$ and $2 \cos^2 \alpha = 1 - \cos^2 \gamma = \cos^2(R - \gamma)$ where γ is the angle with the z-axis (perpendicular incidence). Therefore,

$$\cos(R - \gamma) = \sqrt{2} \cos \alpha$$

Because of

$$\cos \alpha = \cos \alpha_0 + h_1 \frac{\lambda}{d} \quad \text{and} \quad \alpha_0 = R - \gamma_0$$

there is

$$\cos(R - \gamma) = \cos(R - \gamma_0) + h_1 \frac{\lambda}{d} \sqrt{2}$$

that is, the spectra are those of a line grid with the grid constant $\frac{d}{\sqrt{2}}$.

Calculation of the Wavelength λ_m Having the Greatest Intensity

For the velocity distribution in the molecular beam there applies according to Maxwell¹⁴

$$dn = C e^{-\frac{v^2}{\alpha^2}} v^3 dv$$

(α is the most probable velocity in the gas at rest.) According to De Broglie there is

$$\lambda = \frac{h}{mv} \quad \text{or} \quad v = \frac{h}{m\lambda} \quad \text{and} \quad dv = -\frac{h}{m\lambda^2} d\lambda$$

if

$$\lambda_\alpha = \frac{h}{m\alpha}$$

there results

$$dn = C' e^{-\frac{\lambda_\alpha^2}{\lambda^2}} \frac{1}{\lambda^5} d\lambda = C' f(\lambda) d\lambda$$

¹⁴Cf. the remark of A. Einstein in Zeitschrift für Physik 3, 1920, p. 417.

For the wavelength λ_m of greatest intensity, $\frac{dn}{d\lambda} = C'f(\lambda)$ is a maximum, thus

$$\frac{df(\lambda)}{d\lambda} = 0$$

Hence, follows

$$\frac{\lambda_m^2}{\lambda_\alpha^2} = \frac{2}{5} \quad \lambda_m = \lambda_\alpha \sqrt{0.4}$$

Here

$$\lambda_\alpha = \frac{h}{m\alpha} = 30.8 \times 10^{-8} \frac{1}{\sqrt{T_m}} \text{ cm}$$

thus,

$$\lambda_m = 19.47 \times 10^{-8} \frac{1}{\sqrt{T_m}} \text{ cm}$$

The numerical values for the grid constant d have been taken from Landolt-Börnstein; for

$$\text{NaCl: } d = 3.980 \times 10^{-8} \text{ cm}$$

and for

$$\text{LiF: } d = 2.845 \times 10^{-8} \text{ cm}$$

The values of λ_m and λ_m/d for the cases investigated are given in table 5.

With the aid of these values and the equation derived above

$$\tan \vartheta' = (\tan \vartheta_0) \left[1 + \delta \tan \alpha_0 \left(1 + \sqrt{1 - \frac{\tan^2 \vartheta_0}{\tan^2 \alpha_0}} \right) \right]$$

the diffraction angles of λ_m given in the text are calculated. If it is required that the maxima of the curves agree as to position with the diffraction angles calculated for λ_m , the dispersion is presupposed to be constant. For the cross-grid spectra that were investigated, this is always true with sufficient approximation. (Cf. p. 10.) For the line-grid spectra also, the shift of the intensity maximum caused by the variable dispersion is generally small in the cases investigated. The calculation of this shift is, briefly, as follows: Here

$$\cos(R - \gamma) = \cos(R - \gamma_0) \pm \frac{\lambda}{d} \sqrt{2} \quad (h_1 = \pm 1)$$

and

$$dn = C'e^{-\frac{\lambda_\alpha^2}{\lambda^2}} \frac{1}{\lambda^5} d\lambda = C'f(\lambda)d\lambda = C''\phi(\gamma)d\gamma$$

the maximum of this curve lies at

$$\frac{d\phi(\gamma)}{d\gamma} = 0$$

whence there results that

$$2 \frac{\lambda_\alpha^2}{\lambda_m^2} - 5 = \pm \frac{\lambda_m}{d} \frac{\cos(R - \gamma_m)}{\sin^2(R - \gamma_m)}$$

where γ_m is the diffraction angle to which corresponds the greatest intensity and λ_m is the pertaining wavelength. The shift of the intensity maximum calculated from the above (for small shifts an approximation formula was used) always increases the angle $(R - \gamma_m)$ with the crystal surface. For $h = +1$ (diffracted beam between crystal surface and reflected beam), this shift is, for the angle of incidence of 45° , 11° ($R - \gamma_m = 19^\circ$ instead of 8°), but even for an angle of incidence of only 55° , it is not more than 2° , and for the larger angles of incidence, it is still smaller; for $h = -1$ (diffracted beam on the other side of the reflected beam) it is always at most 1° or smaller.

TABLE 1

Diffraction angle, deg	Deflection, cm	Tilt angle, deg	Diffraction angle, deg	Deflection, cm	Tilt angle, deg
0	7.4	0			
-4	2.55	0	4	2.2 2.25 <2.2	0 $1\frac{1}{2}$ 1
-6	3.6 3.1	$1\frac{1}{2}$ 1	6	3.05 3.25 <3.2	$1\frac{1}{2}$ 1 $1\frac{1}{2}$
-8	4.45 4.55 <4.5	$1\frac{1}{2}$ 1 $1\frac{1}{2}$	8	5.13 5.55 5.2	1 $1\frac{1}{2}$ 2
-10	5.35 <5.3	$1\frac{1}{2}$ 2	10	5.0 5.42 5.25 4.7	$1\frac{1}{2}$ 2 $2\frac{1}{2}$ 3
-11	5.4 <5.4	$1\frac{1}{2}$ 2			
-12	5.2 <5.2	$1\frac{1}{2}$ 2	12	4.0 <4.0	$2\frac{1}{2}$ 2.3
-14	3.45 4.4 4.1	$1\frac{1}{2}$ 2 $2\frac{1}{2}$	14	2.9 <2.9 2.6	$3\frac{1}{2}$ 3 $4\frac{1}{2}$
-16	3.15 3.0	$3\frac{1}{2}$ 4			

TABLE 2

Beam temperature	Diffraction angle	
	Order 01	Order 02
100° K	$14\frac{1}{2}^{\circ}$	$29\frac{1}{2}^{\circ}$
290	$8\frac{1}{2}$	17
580	6	12

TABLE 3

Gas	Beam temperature	Location of maximum		Curve
		Calculated	Measured	
He {	290° K	$11\frac{3}{4}^{\circ}$	12°	XVI
	580	$8\frac{1}{2}$	$8\frac{3}{4}$	XVII
H ₂ {	290	$16\frac{3}{4}$	$14\frac{1}{2}$	XVIII
	580	$11\frac{3}{4}$	12	XIX

TABLE 4

Gas	Beam temperature	Location of maximum		Curve
		Calculated	Measured	
He	100° K	21°	$15\frac{1}{2}$	XXI
	180	$15\frac{1}{2}$	$14\frac{1}{2}$	XXII
	290	12	$11\frac{1}{2}$	XXIII
	590	$8\frac{3}{4}$	9	XXIV
H ₂	290	17	17	XXV
	580	12	11	XXVI

TABLE 5

Temperature, °K	He = 4.00			H ₂ = 2.016		
	$\lambda_m \times 10^8$ cm	$\frac{\lambda_m}{d}$ for NaCl	$\frac{\lambda_m}{d}$ for LiF	$\lambda_m \times 10^8$ cm	$\frac{\lambda_m}{d}$ for NaCl	$\frac{\lambda_m}{d}$ for LiF
100	0.974	0.2446	0.3423	1.371	0.3450	0.4828
180	.727	-----	.255	-----	-----	-----
295	.570	.1432	.2003	.805	.2024	.2830
590	.405	.1018	.1424	.570	.1432	.2003

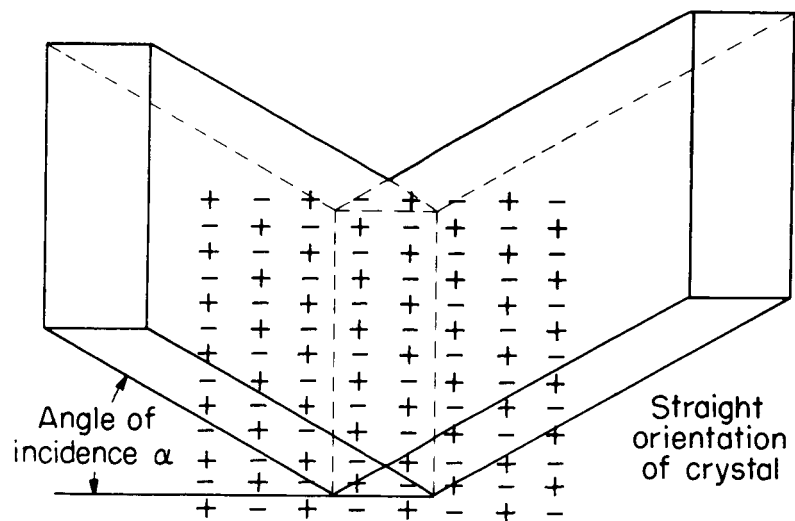


Figure 1.

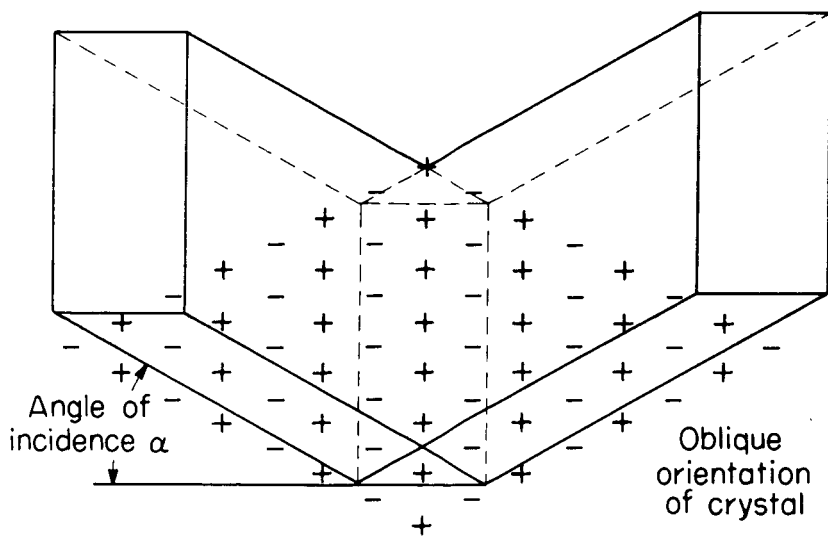


Figure 2.

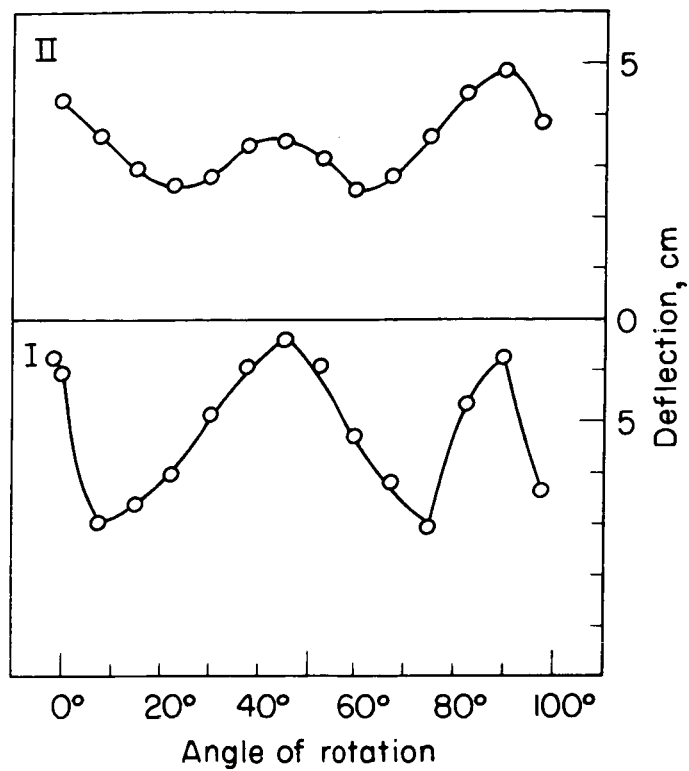


Figure 3.- Dependence of the reflectivity on the orientation of the crystal.

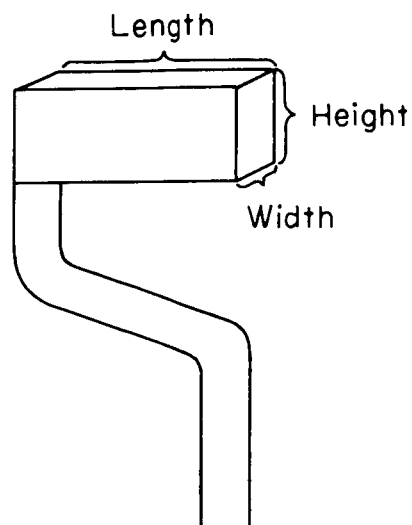


Figure 4.- Receiver.

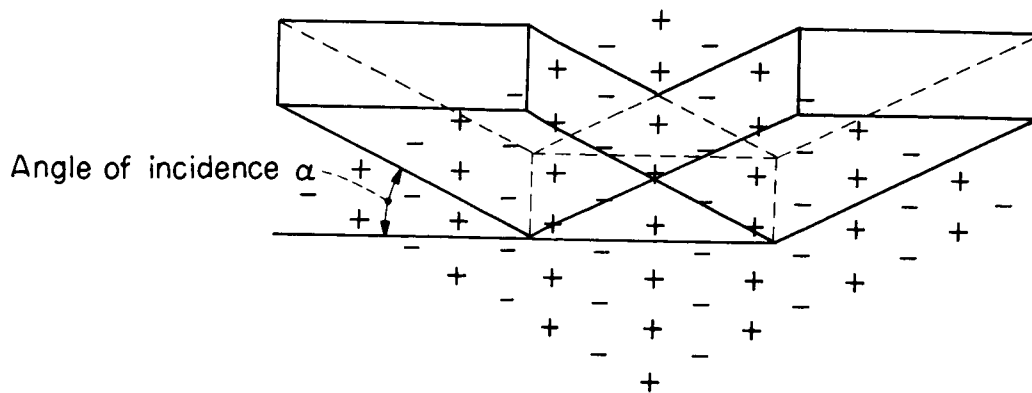


Figure 5.- Beam incident "up-end."

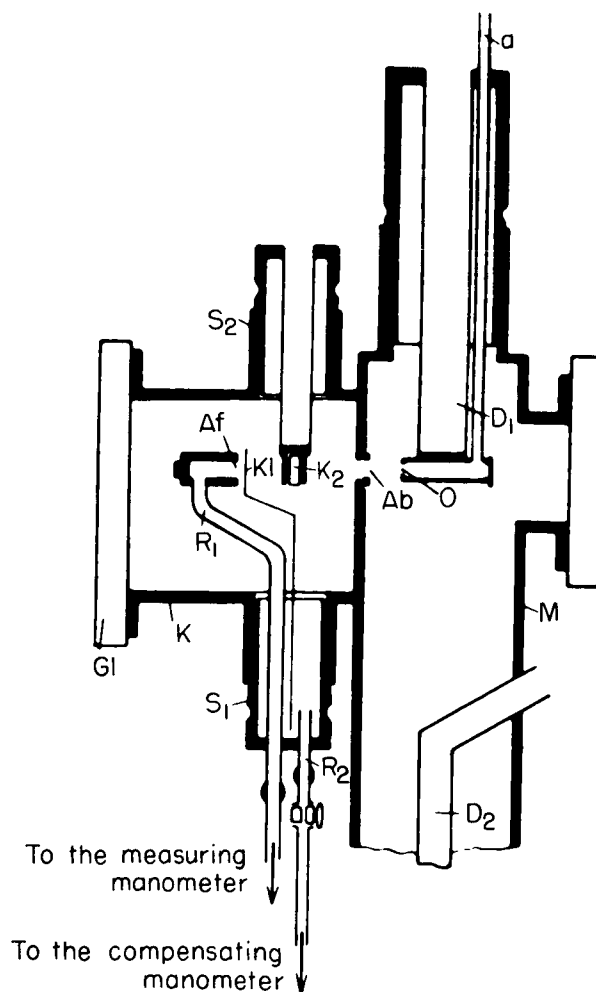


Figure 6.- Old arrangement. O furnace slit; Ab image-forming slit; Af receiver; K₂ crystal; K₁ flap valve; a supply tube for gas; and D₁ cooling vessel. For the remaining symbols, see also U.z.M. no. 11, l.c.

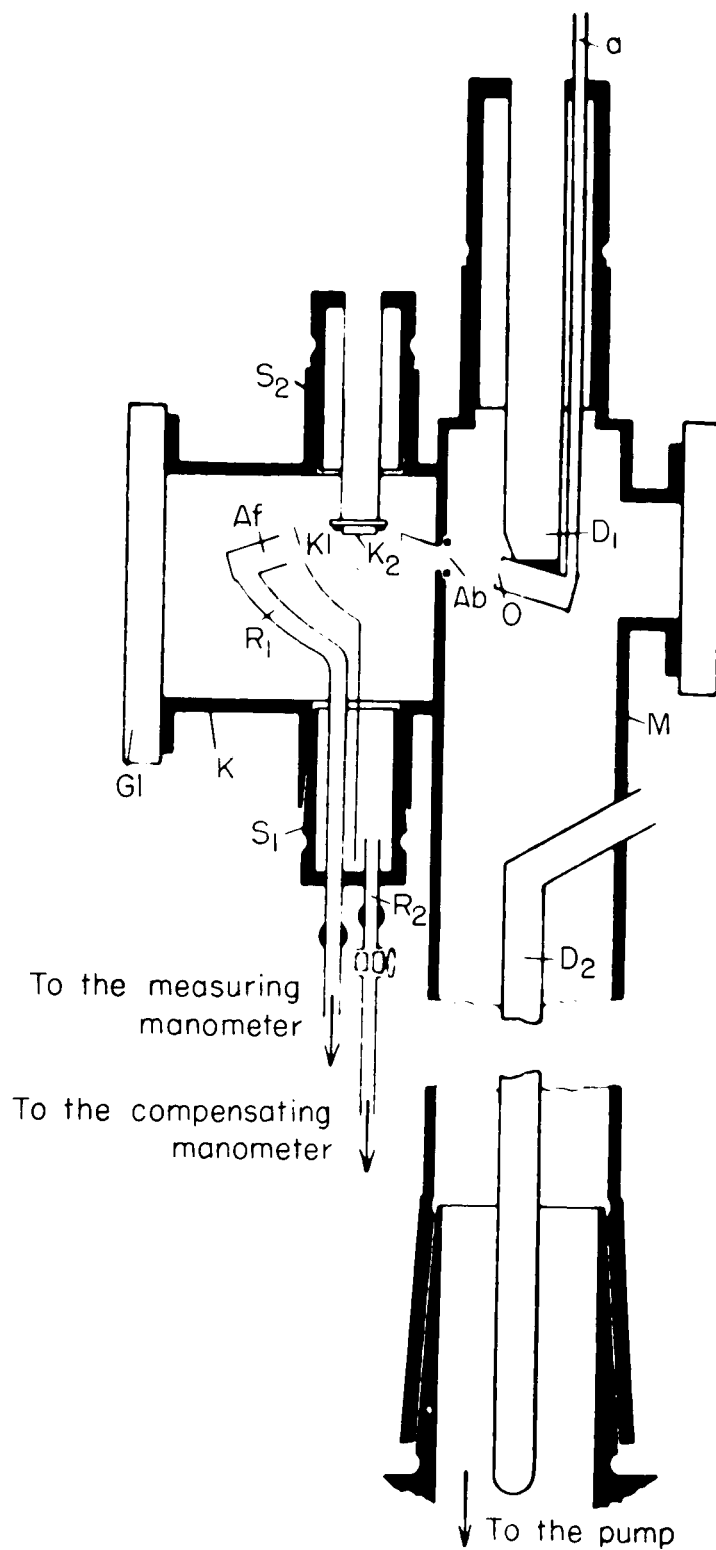


Figure 7.- New arrangement.

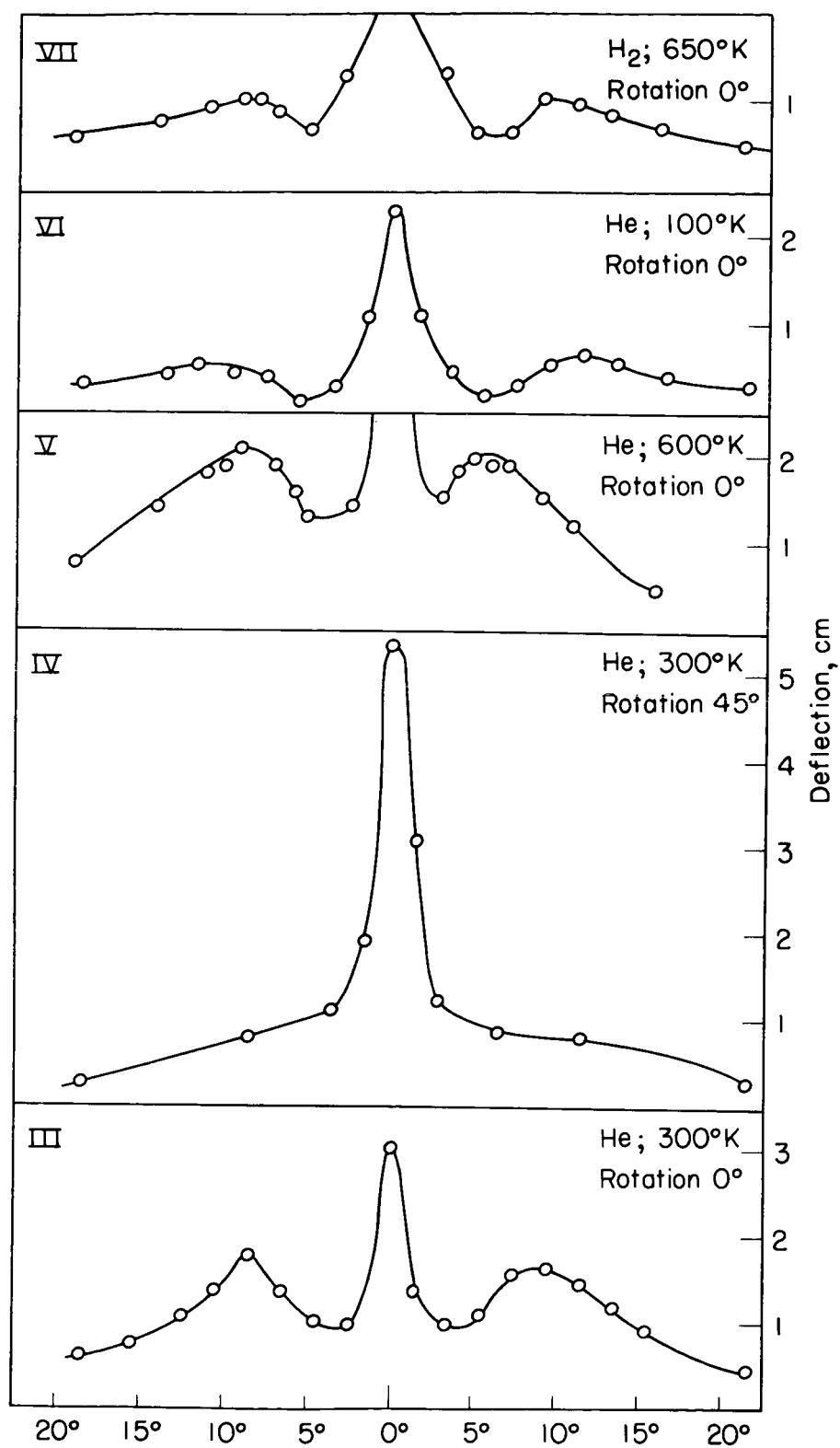


Figure 8.- Diffraction of He and H₂ from NaCl.

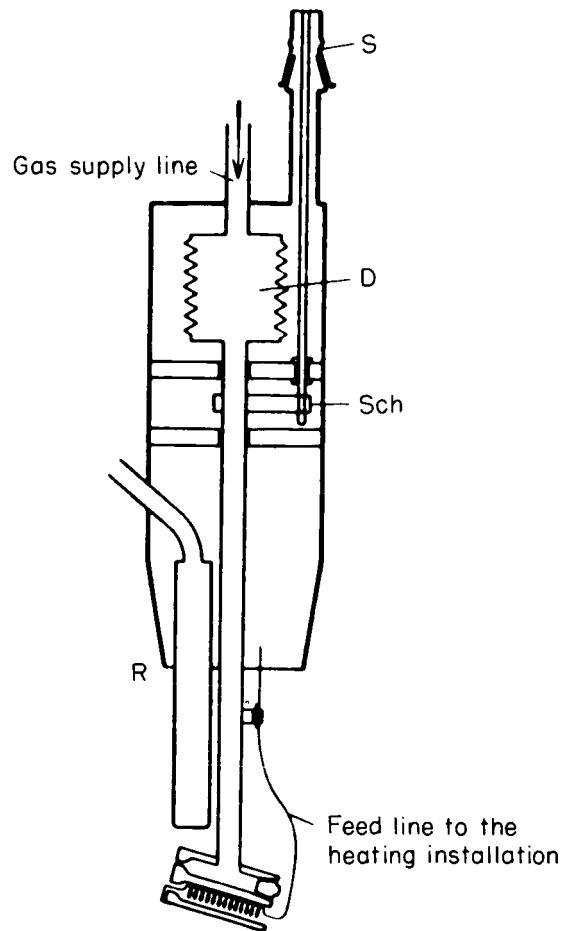


Figure 9.- Adjustable furnace slit.

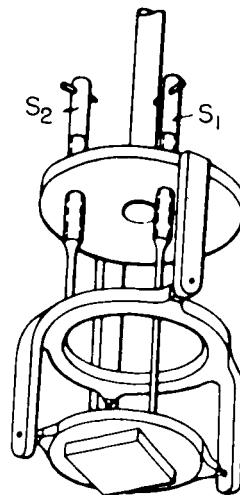


Figure 10.- Crystal support.

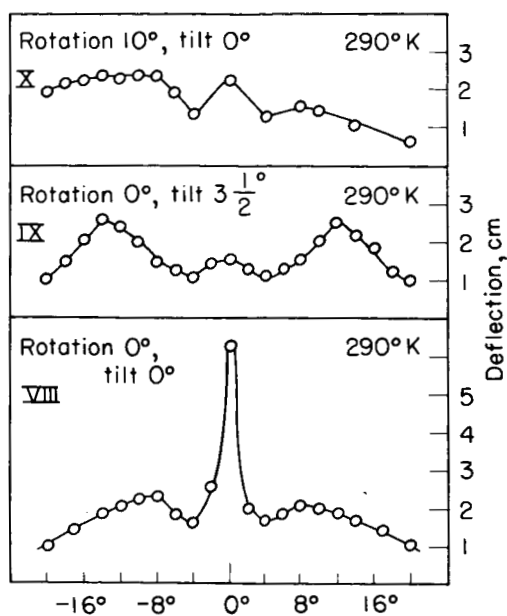


Figure 11.- Diffraction of He from NaCl.

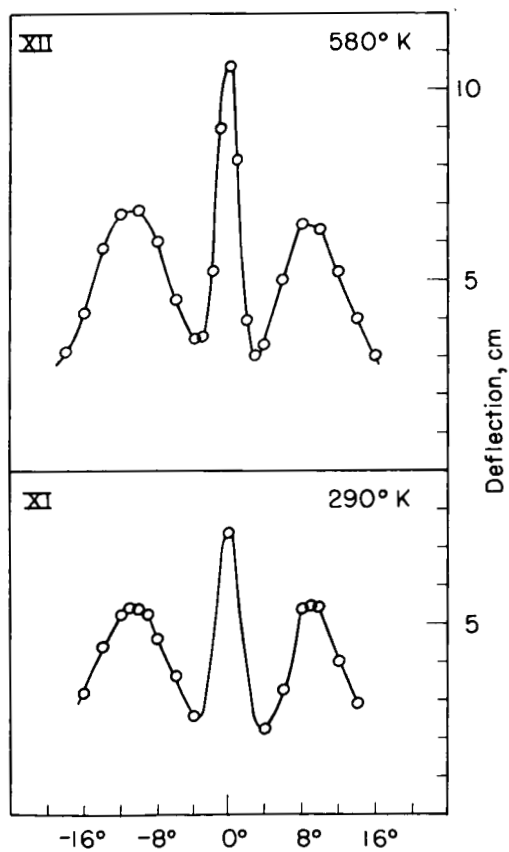


Figure 12.- Diffraction of He from NaCl.

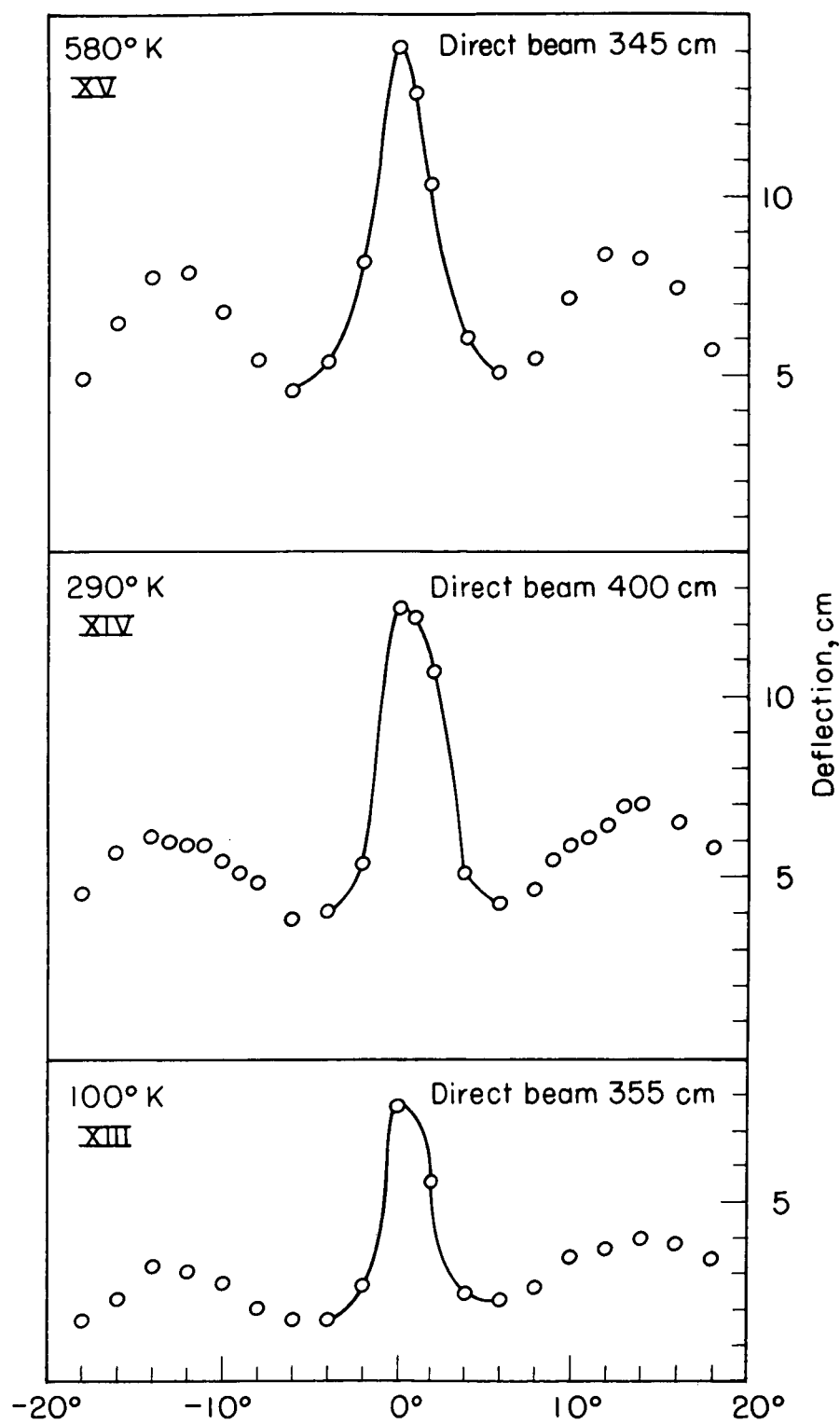


Figure 13.- Diffraction of He from NaCl.

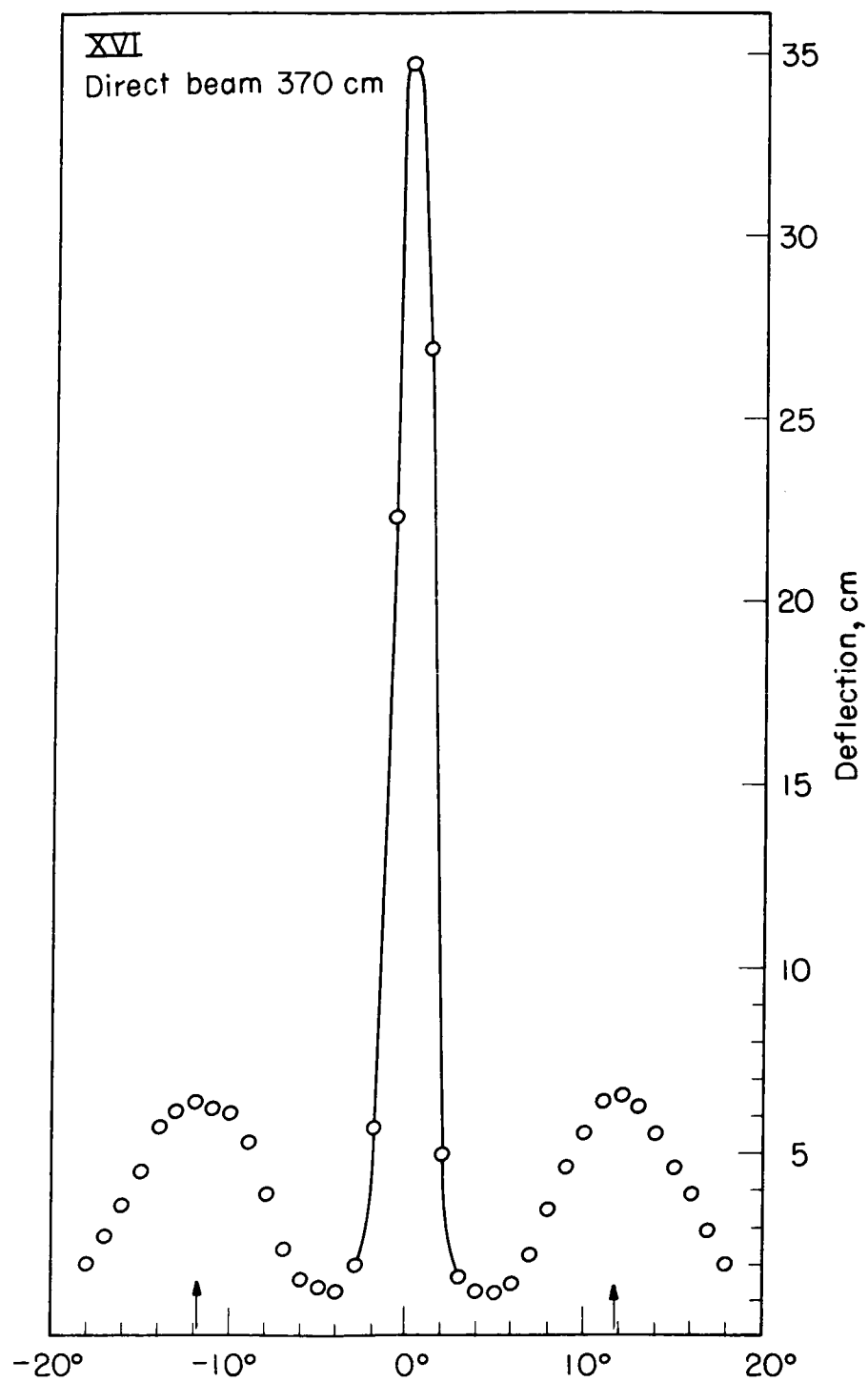


Figure 14.- Diffraction of He from LiF, 295° K; angle of incidence $11\frac{1}{2}^\circ$.

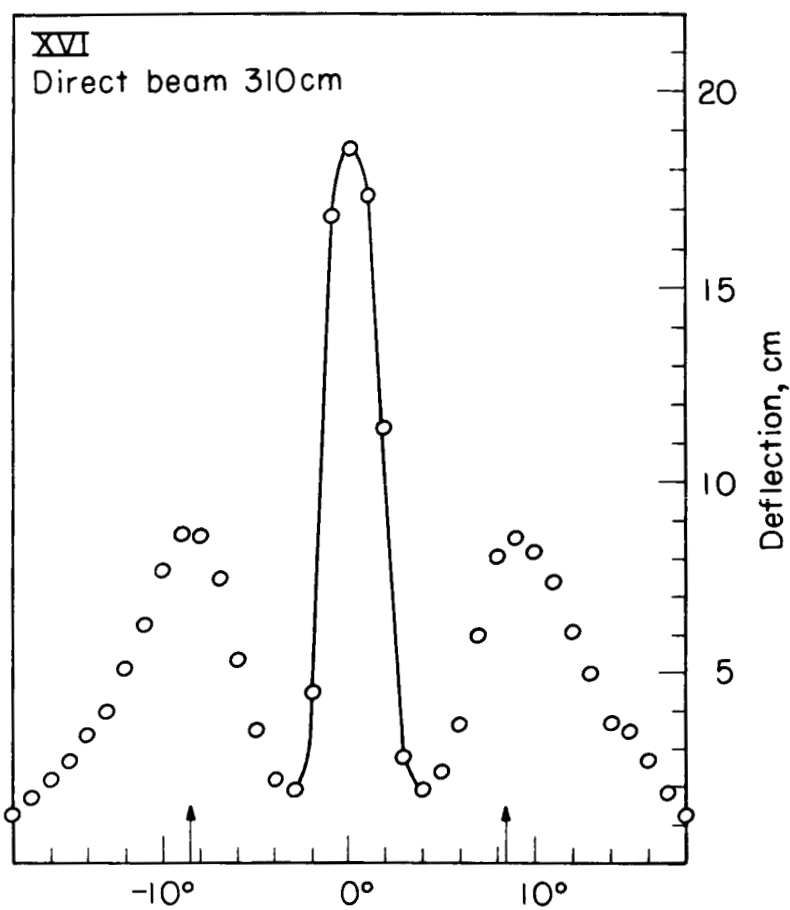


Figure 15.- Diffraction of He from LiF, 580° K; angle of incidence $11\frac{1}{2}^\circ$.

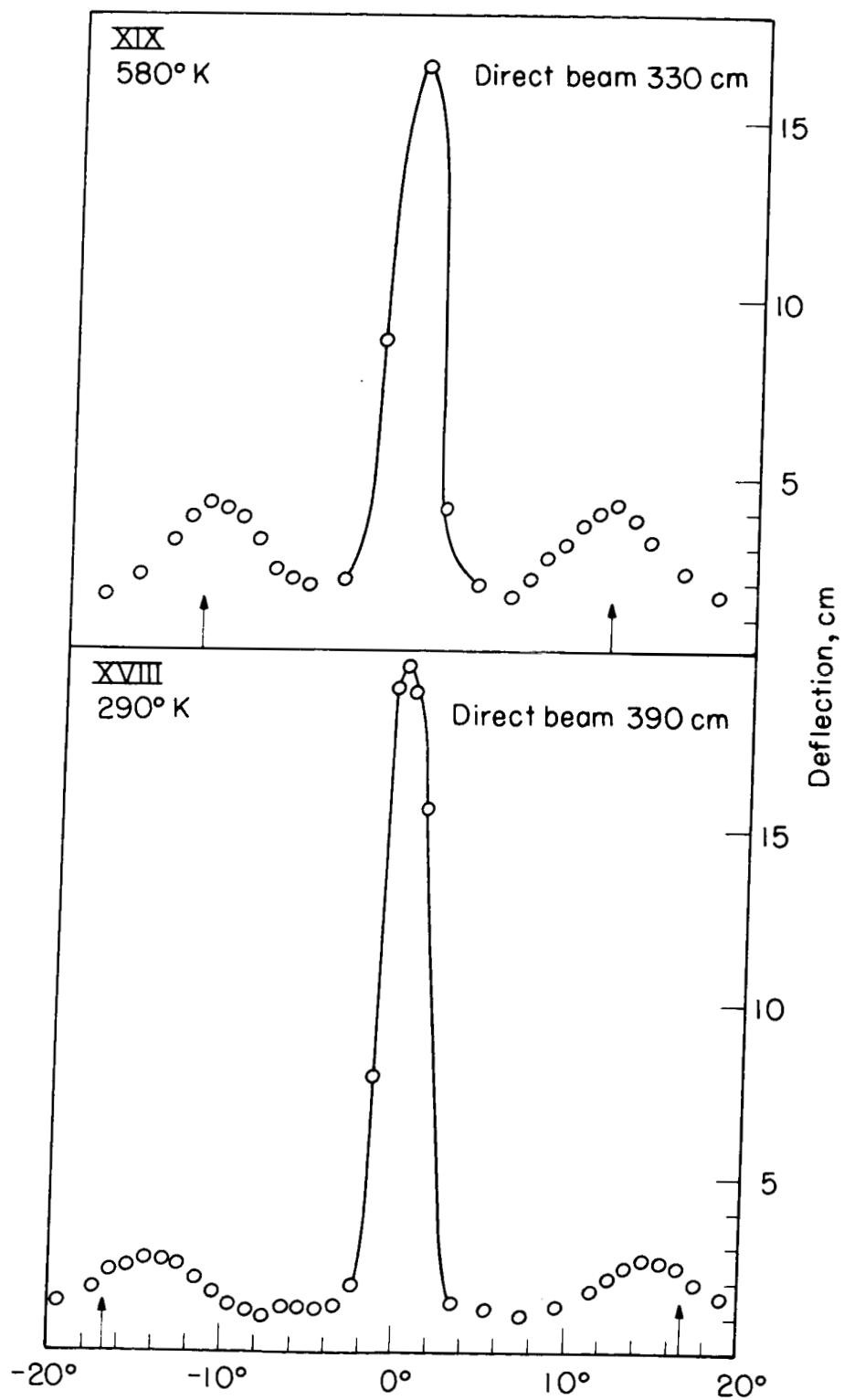


Figure 16.- Diffraction of H_2 from LiF ; angle of incidence $11\frac{1}{2}^\circ$.

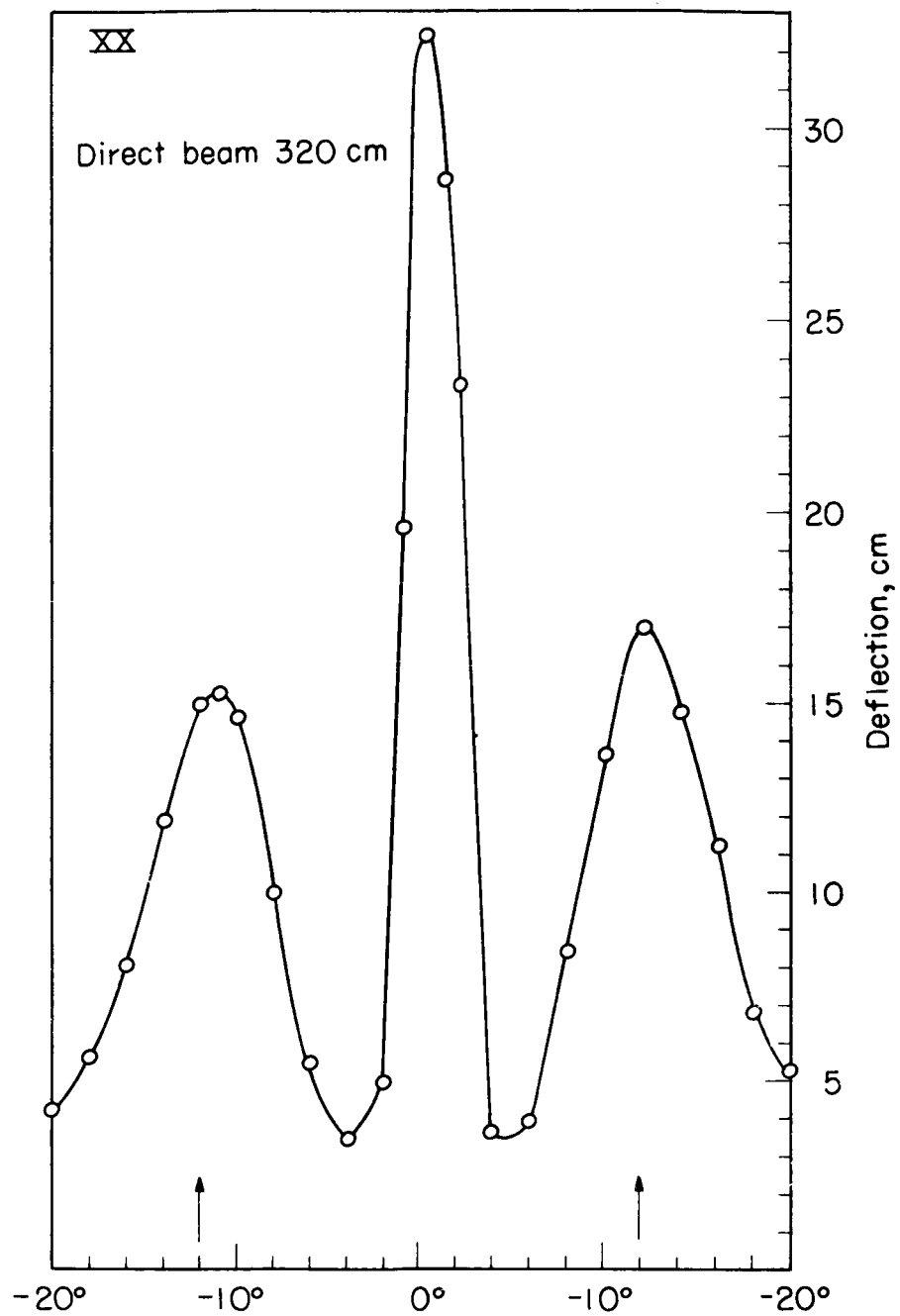


Figure 17.- Diffraction of He from LiF, 290° K; angle of incidence $18\frac{1}{2}^\circ$.

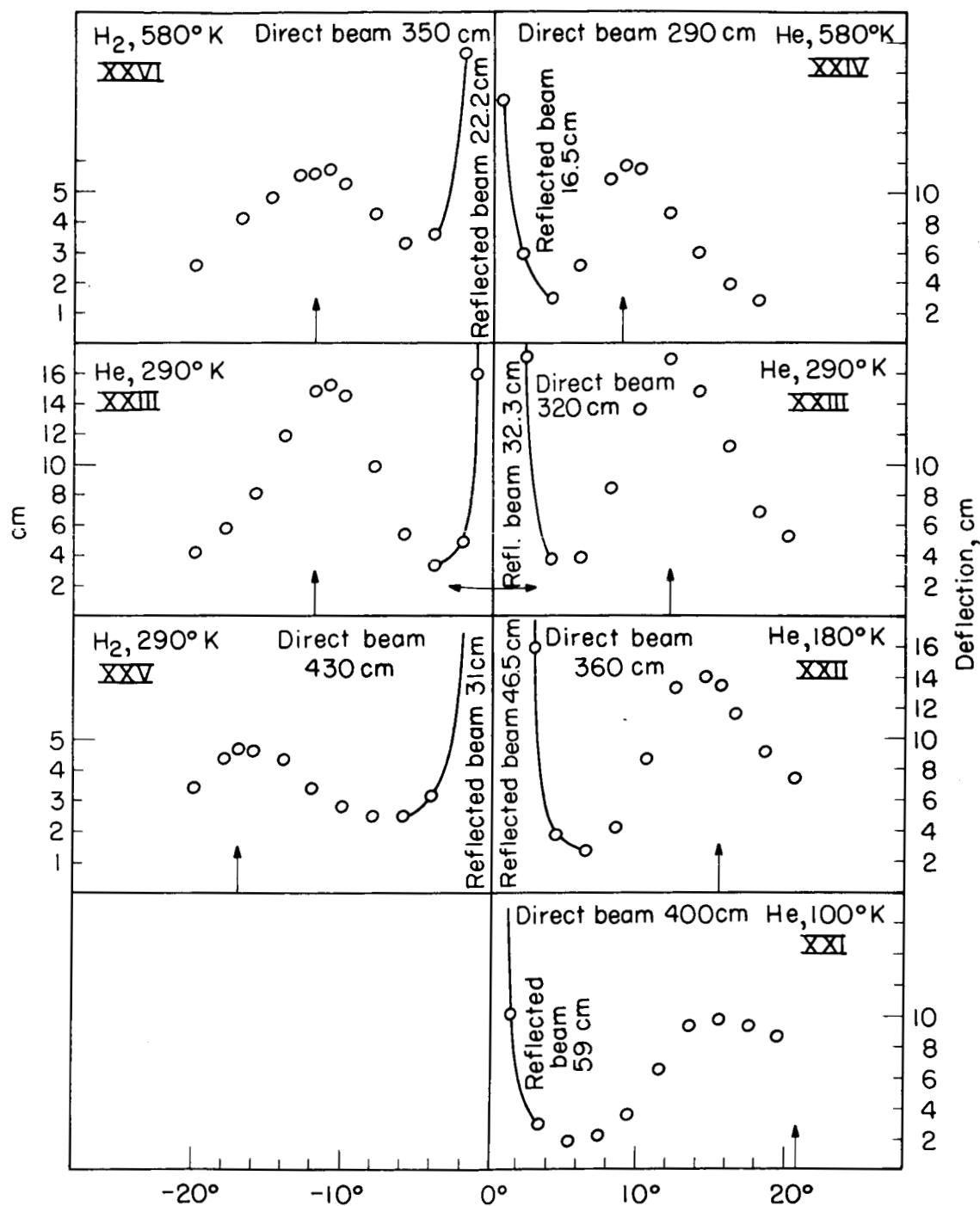


Figure 18.- Diffraction of He and H₂ from LiF; angle of incidence $18\frac{1}{2}^\circ$.

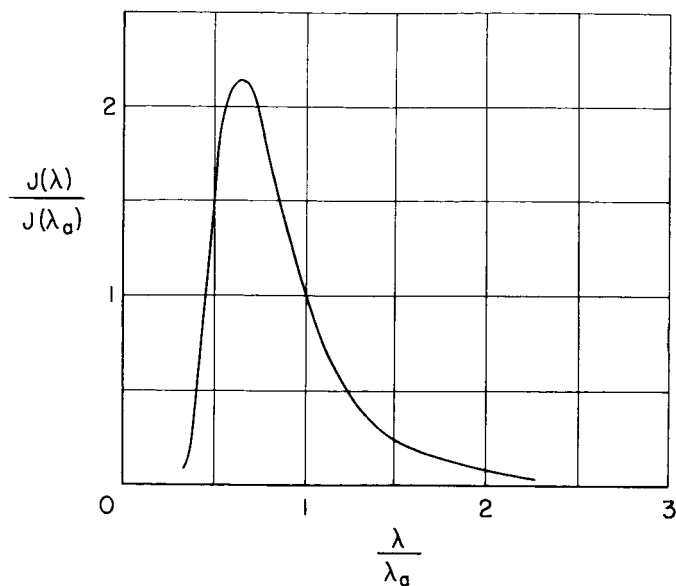


Figure 19.- Maxwell distribution of the De Broglie wave lengths in the molecular beam.

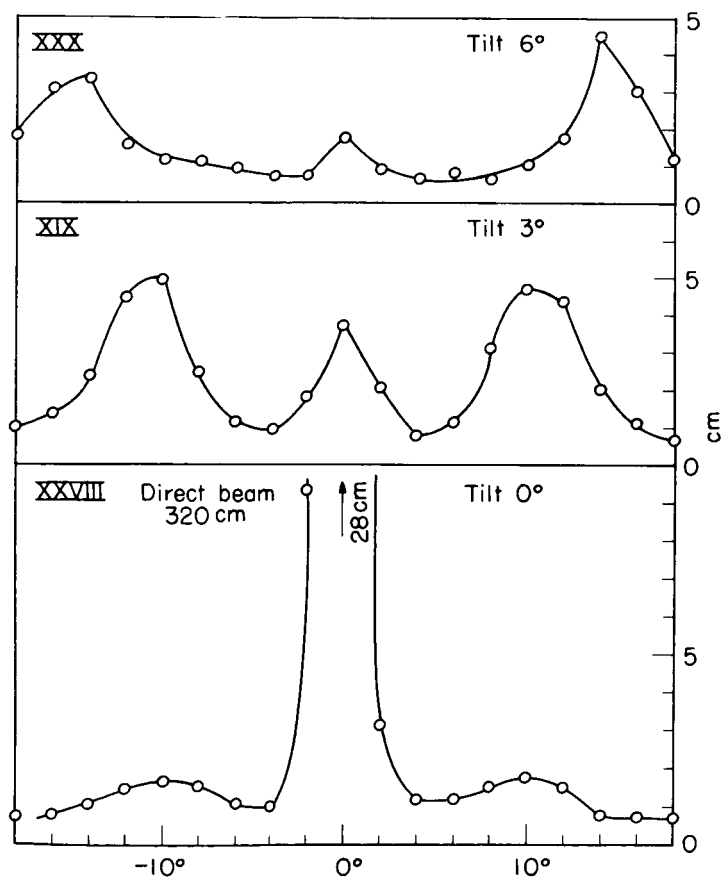


Figure 20.- Diffraction of He from LiF with constant tilt.

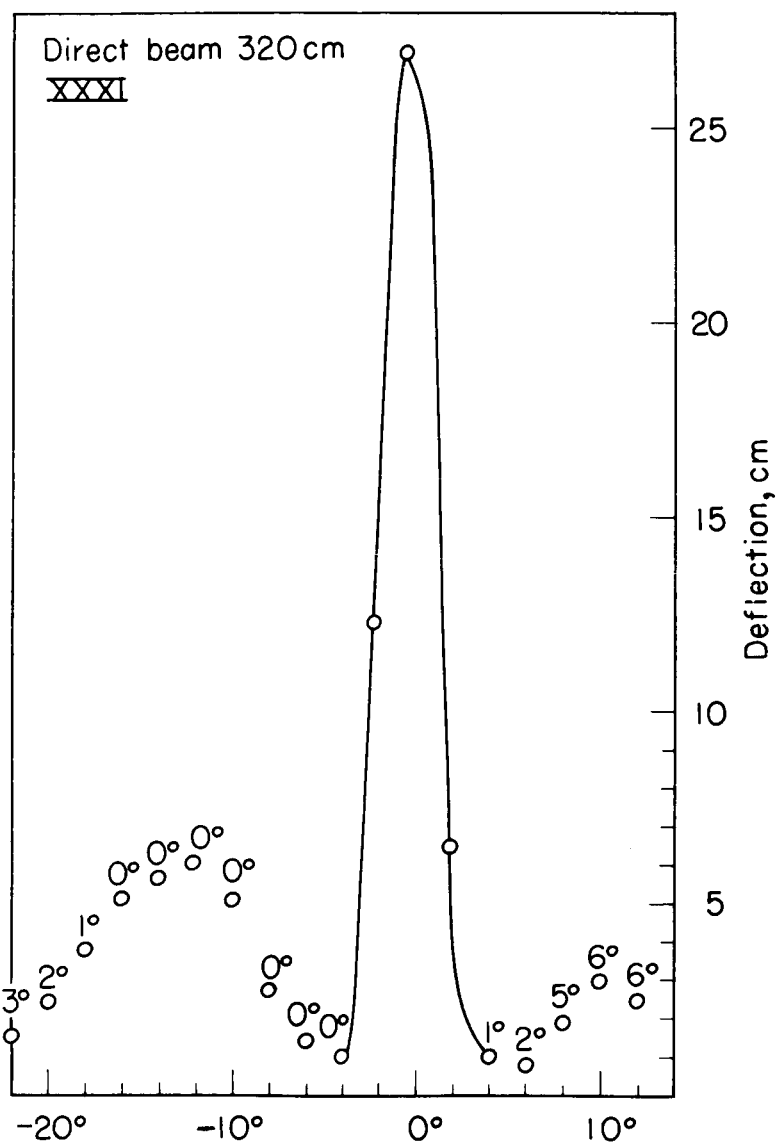


Figure 21.- Diffraction of He from LiF with rotated crystal;
rotation 8°.

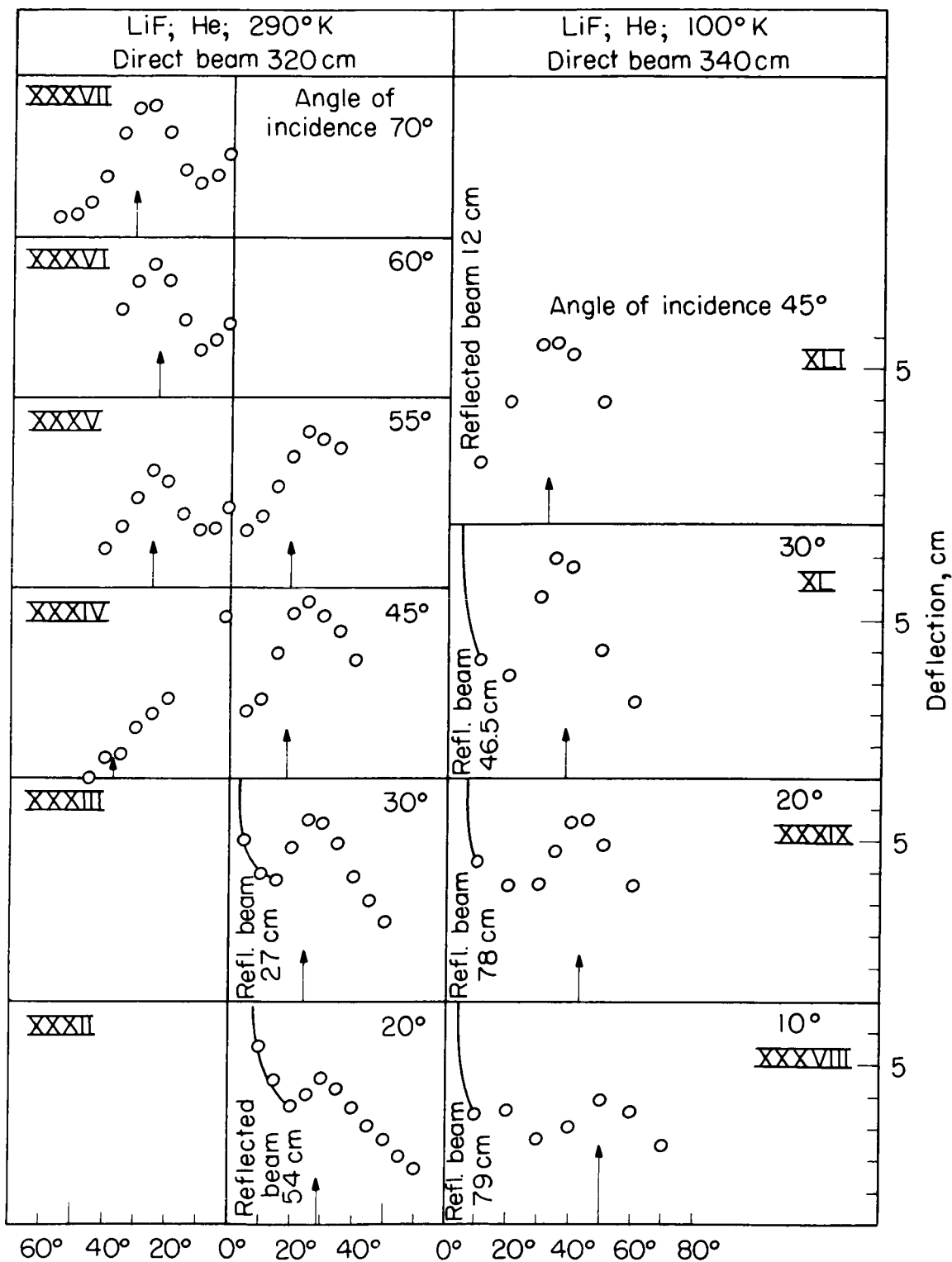


Figure 22.- Diffraction spectra in the plane of incidence.

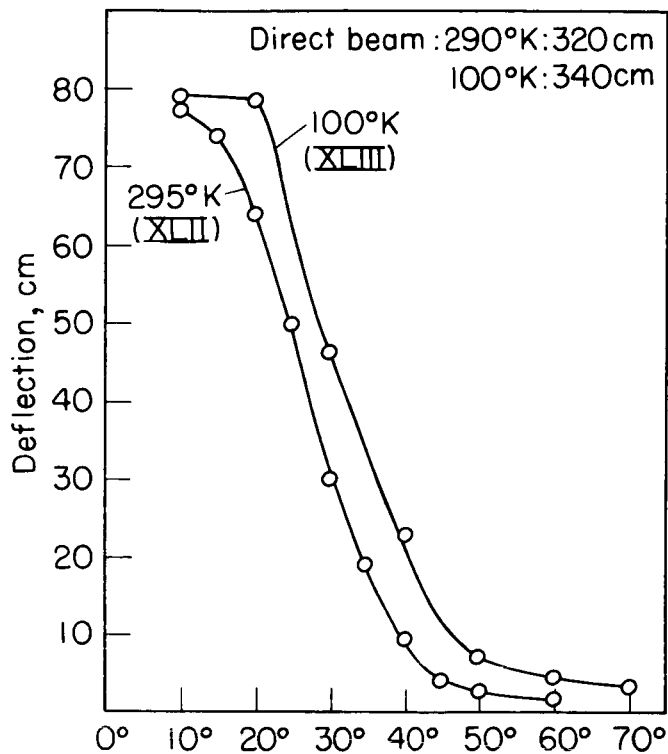


Figure 23.- Reflectivity of LiF for He.

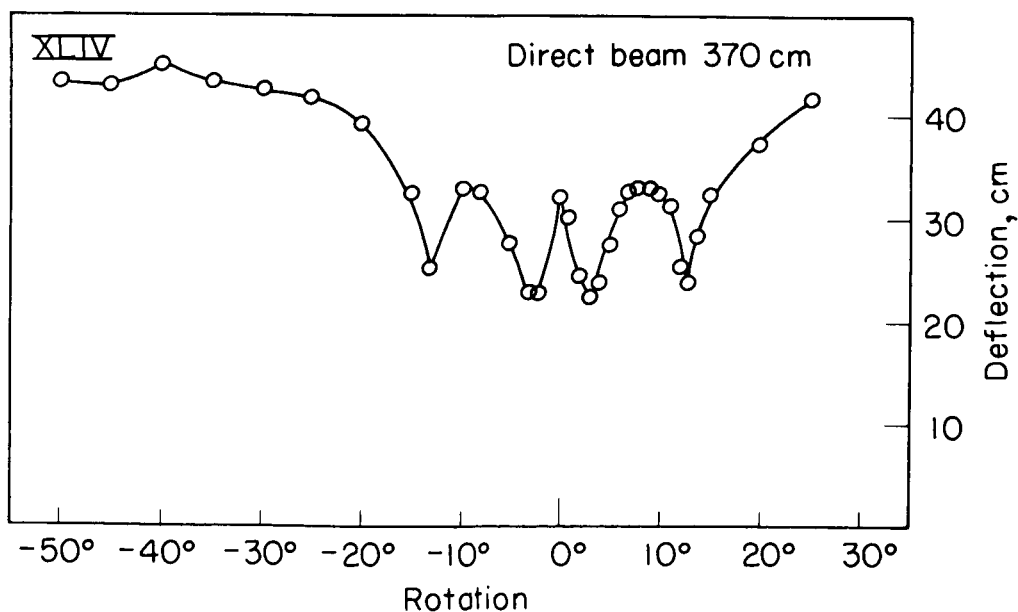


Figure 24.- Dependence of the reflectivity on the orientation of the crystal.

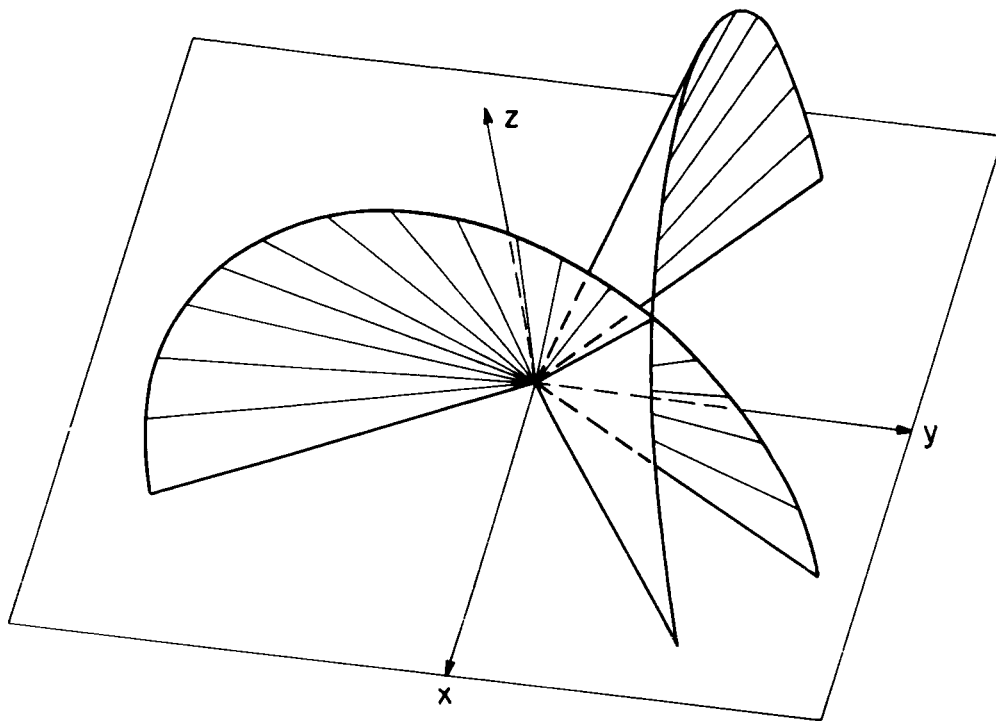


Figure 25.- Construction of the cross-grid spectra.

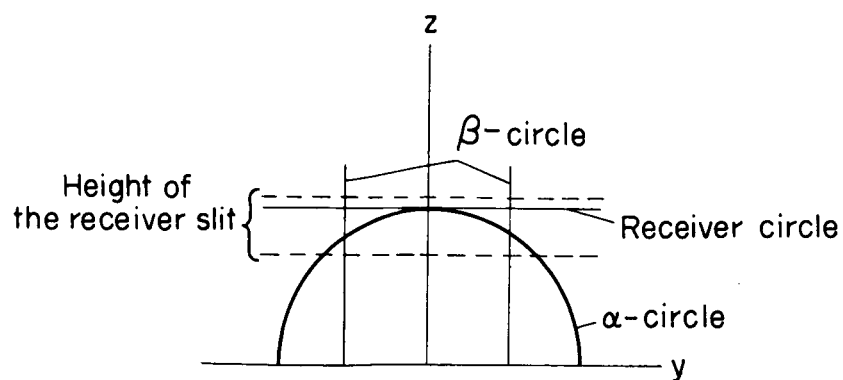


Figure 26.- Parallel projection parallel to the x -axis onto the y, z -plane.

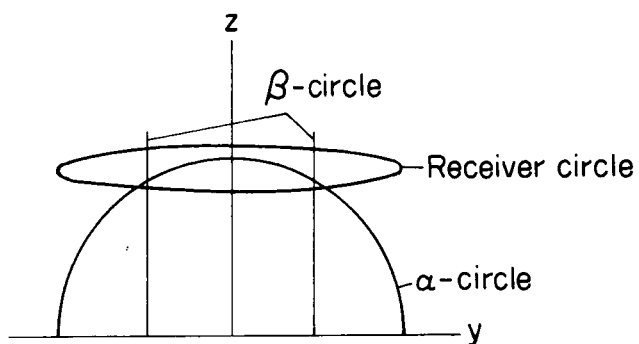


Figure 27.- Parallel projection for a tilted crystal.

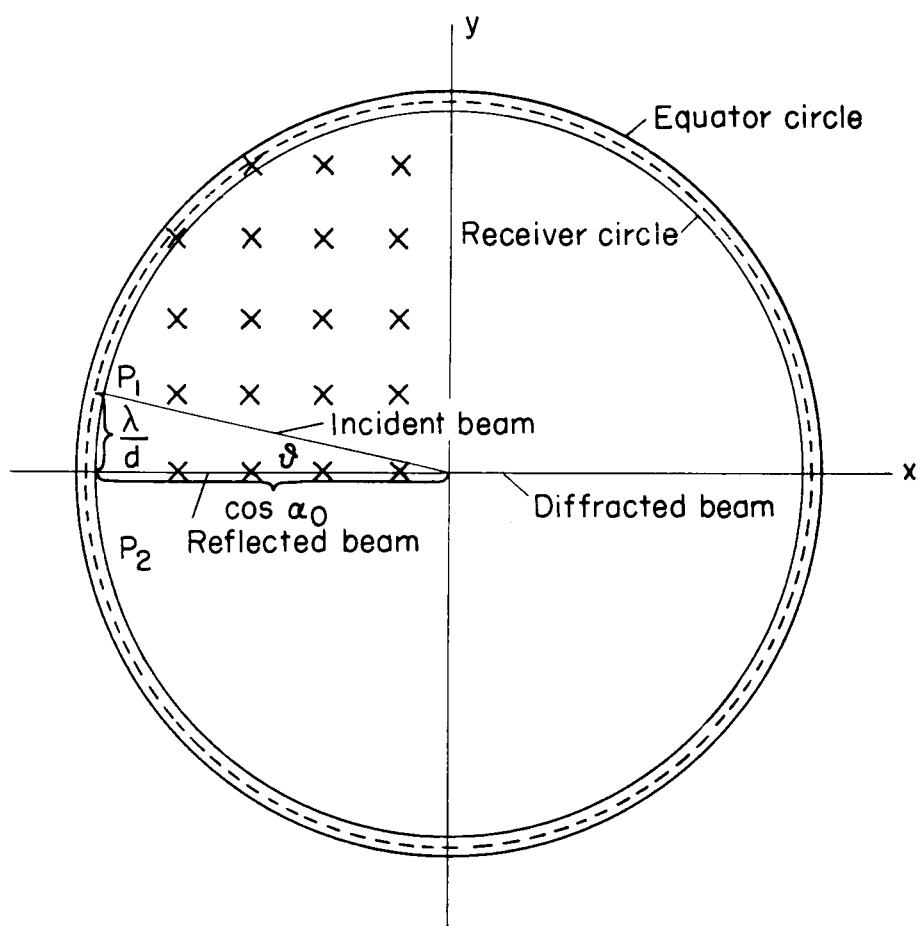


Figure 28.- Parallel projection parallel to the z -axis onto the x,y -plane; reciprocal grid.

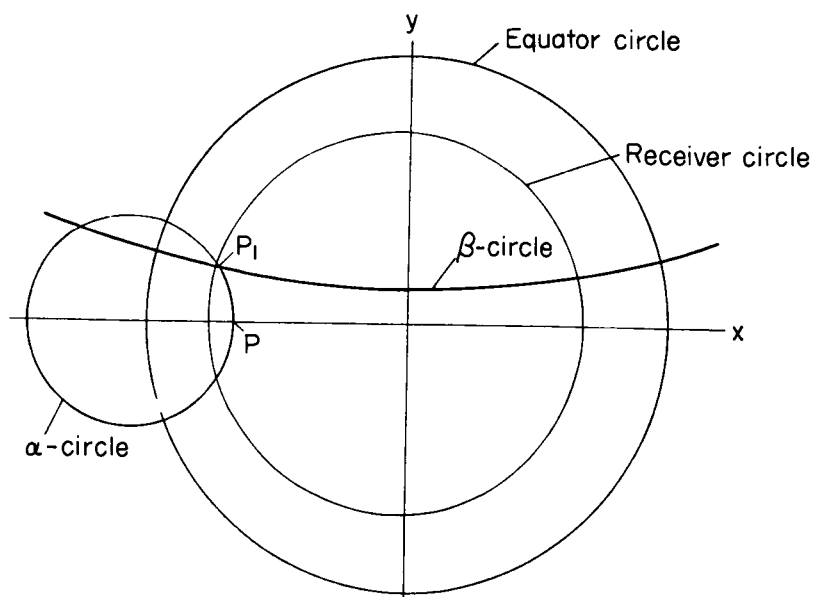


Figure 29.- Construction of the diffracted beam in stereographic projection for a tilted crystal.

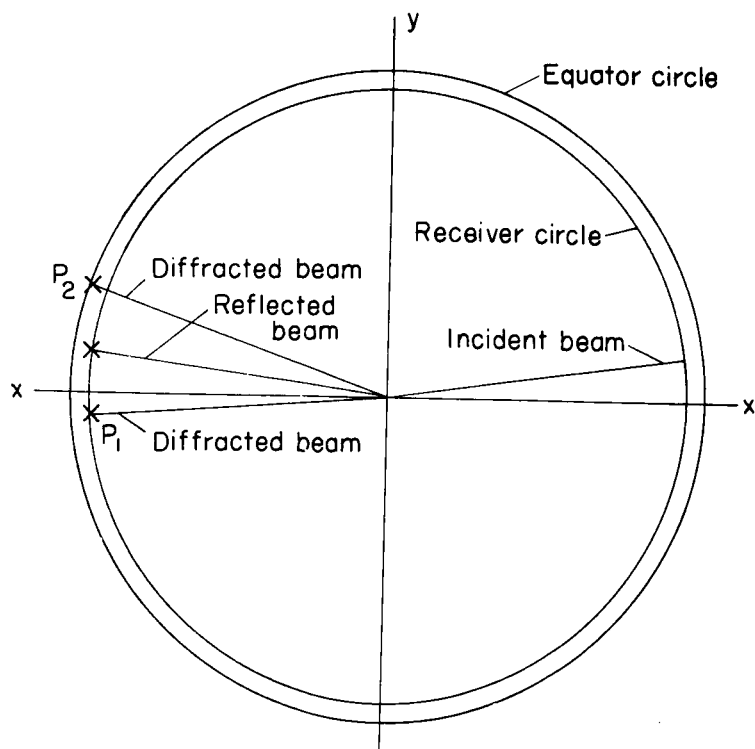


Figure 30.- Parallel projection for a rotated crystal.